Y.0 EXAMPLE CALCULATIONS—INITIAL CORRELATION TEST

This section presents several examples of initial correlation test calculations. For each of the examples, a brief description of the correlation test scenario is provided, followed by a summary and discussion of the results of the calculations. For the first example, all of the equations used to evaluate the data and some sample calculations are presented. The calculations for all of the examples were performed using the Correlation Test Calculation Spreadsheet, which is described in Section X. Appendix A includes the entire spreadsheet output for each of the examples.

Y.1 EXAMPLE 1: HOW DO I DEVELOP AN INITIAL CORRELATION?

Y.1.1 Correlation Test Scenario

A correlation test is performed on a boiler at Facility A. The boiler is equipped with an in situ PM CEMS that responds to PM concentrations at actual conditions. The emission limit, when converted to units that are consistent with the PM CEMS sampling conditions, is 20.0 mg/acm. Twenty-four test runs are performed using a single Method 5 sampling train. The facility determines that four of the test runs (Runs 7, 8, 9, and 18) were suspect because of problems with the sampling train. Table Y-1 summarizes the correlation test data. Figure Y-1 shows a plot of the data.

Table Y-1. Summary of Correlation Test Data for Facility A

Run no.	PM CEMS response, mA	PM concentration, mg/acm
1	4.90	2.1
2	7.60	2.9
3	7.80	5.1
4	9.20	4.8
5	10.10	2.9
6	11.06	6.3
7	13.40	0.2
8	16.10	0.6
9	15.80	0.3
10	12.00	8.4

(continued)

Table Y-1. (continued)

Run no.	PM CEMS response, mA	PM concentration, mg/acm
11	18.20	8.4
12	15.70	4.9
13	13.90	6.3
14	19.70	13.0
15	21.02	9.1
16	22.30	13.9
17	24.10	8.7
18	25.90	0.6
19	24.70	12.6
20	26.40	15.9
21	29.08	16.2
22	29.30	13.5
23	30.50	15.9
24	33.10	18.5

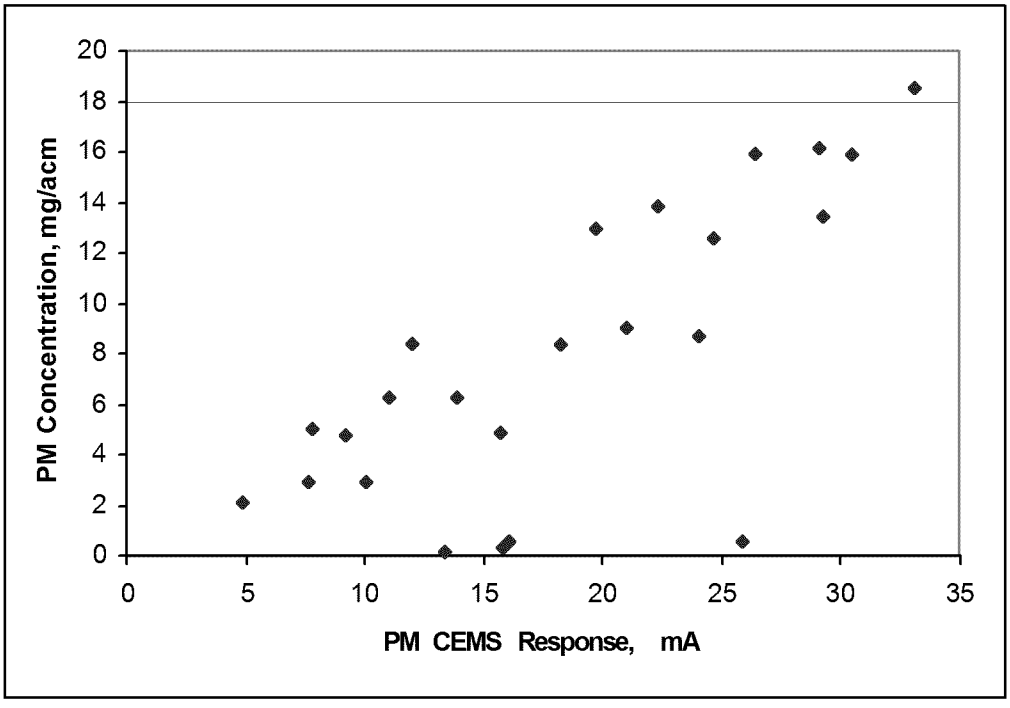

Figure Y-1. Plot of the correlation test data for Facility A.

Must I use all of the test data to develop the correlation?

Section 8.6(3) of PS-11 requires a minimum of 15 valid test runs for developing the correlation, and Section 8.6(3)(iii) allows you to discard up to 5 test runs without explanation. Additional test runs can be discarded if the basis for rejecting the runs is specified in the manual test method, in PS-11, or in the quality assurance plan for the source. In this example, an additional nine test runs were performed, so it is not necessary to use all of the test data. Five of the runs can be discarded without explanation. As indicated in Figure Y-1, the results from test Runs 7, 8, 9, and 18 appear to be outliers. If the four outliers are discarded, there are 20 runs remaining, which satisfies the minimum requirement of 15 runs for developing the initial correlation. Note that Section 8.6(3)(ii) of PS-11 still requires you to report all of the data, even for those runs that are rejected.

How do I develop a linear correlation curve from the data?

To develop a linear correlation model, follow the procedures described in Section 12.3(1) of PS-11. The form of the linear model is provided in Equation 11-3 of PS-11:

$$\hat{y} = b_0 + b_1 x$$

where

- $\hat{y}$ = the predicted PM concentration
- b_0 = the intercept for the correlation curve
- b_1 = the slope of the correlation curve
- x = the PM CEMS response value.

To begin, the mean PM CEMS response ($\bar{x}$) and mean PM concentration ($\bar{y}$) are calculated as follows:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{20}(4.9 + 7.6 + 7.8 + \ldots + 33.1) = 18.53$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{20}(2.1 + 2.9 + 5.1 + \ldots + 18.5) = 9.47$$

The variables S_{xx} and S_{xy}, which are presented in Equation 11-7 of PS-11, are calculated next as follows:

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 = \left((4.9 - 18.53)^2 + (7.6 - 18.53)^2 + \ldots + (33.1 - 18.53)^2\right) = 1{,}442.2$$

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) = (4.9 - 18.53)(2.1 - 9.47) + (7.6 - 18.53)(2.9 - 9.47) + \ldots + (33.1 - 18.53)(18.5 - 9.47) = 785.2$$

The slope of the correlation curve (b_1) is determined by Equation 11-6 of PS-11:

$$b_1 = \frac{S_{xy}}{S_{xx}} = \frac{7852}{1{,}4422} = 0544$$

The intercept of the correlation curve is determined using Equation 11-4 of PS-11:

$$b_0 = \bar{y} - b_1 \square \bar{x} = 947 - (0544 \times 1853) = -0621$$

The resulting equation for the correlation curve is as follows:

$$\text{í} = !0.621 + 0.544x$$

<u>How do I determine the linear correlation coefficient?</u>

The linear correlation coefficient (r) is calculated using the parameters S_L and S_y. The parameter S_L is defined in Equation 11-9 as follows:

$$S_L = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}\left(\$_i - y_i\right)^2}$$

To calculate S_L, the linear correlation equation must be used to determine the predicted PM concentration (í) for each test run. For example, for Run 3, the PM CEMS response is 7.8 and the predicted PM concentration is

$$\$ = -0621 + 0544\,(78) = 3626$$

Table Y-2 shows the results of the calculations of í. Using these values, the calculation for S_L is as follows:

$$S_L = \sqrt{\frac{1}{20-2}\left(\left(2047 - 21\right)^2 + \left(3517 - 29\right)^2 + \left(3626 - 51\right)^2 + \text{K} + \left(17401 - 185\right)^2\right)} = 1910$$

Table Y-2. Predicted PM Concentrations (í) for Linear Correlation

Run no.	PM CEMS response, mA	PM concentration, mg/acm	Predicted PM concentration (í)[a]
1	4.9	2.1	2.047
2	7.6	2.9	3.517
3	7.8	5.1	3.626
4	9.2	4.8	4.388
5	10.1	2.9	4.878

(continued)

Table Y-2. (continued)

Run no.	PM CEMS response, mA	PM concentration, mg/acm	Predicted PM concentration (í)[a]
6	11.06	6.3	5.401
10	12	8.4	5.913
11	18.2	8.4	9.289
12	15.7	4.9	7.928
13	13.9	6.3	6.947
14	19.7	13	10.105
15	21.02	9.1	10.824
16	22.3	13.9	11.521
17	24.1	8.7	12.501
19	24.7	12.6	12.828
20	26.4	15.9	13.753
21	29.08	16.2	15.213
22	29.3	13.5	15.332
23	30.5	15.9	15.986
24	33.1	18.5	17.401

[a] Calculated using the correlation equation: í = -0.621 + 0.544*x*.

The parameter S_y is calculated using Equation 11-15 of PS-11 as follows:

$$S_y = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}{n-1}} = \sqrt{\frac{(2.1-9.47)^2 + (2.9-9.47)^2 + (5.1-9.47)^2 + \mathrm{K} + (18.5-9.47)^2}{20-1}} = 5.095$$

With the values for S_L and S_y, the correlation coefficient is calculated using Equation 11-14 as follows:

$$r = \sqrt{1 - \frac{S_L^{\,2}}{S_y^{\,2}}} = \sqrt{1 - \frac{(1910)^2}{(5095)^2}} = 0927$$

The correlation coefficient (0.927) is greater than 0.85 and, therefore, satisfies PS-11.

Note: For data sets that show little to no correlation, the value for S_L can be greater than S_y. In such cases, the correlation coefficient (r) calculated using Equation 11-14 is undefined because it entails determining the square root of a negative number. Even though the correlation coefficient cannot be determined, it can be assumed that the value for r does not meet the criterion of 0.85 specified by PS-11.

How do I calculate the confidence interval half range percentage for the linear correlation?

The half range for the 95 percent confidence interval (CI) for the predicted PM concentration at the mean value of x is determined using Equation 11-8 of PS-11:

$$CI = t_{df,\,1-a/2} \cdot S_L \cdot \sqrt{\frac{1}{n}}$$

The variable $t_{df,\,1-a/2}$ is obtained from a table of *t*-statistic values. The degrees of freedom (df) equals n-2, and the value for the *t* statistic for this example is 2.101. With this value and the value for S_L calculated above, CI is calculated as follows:

$$CI = 2.101(1.91)\sqrt{\frac{1}{20}} = 0.897$$

The emission limit in this example is 20.0 mg/acm. To determine CI as a percentage of the emission limit, Equation 11-10 is used as follows:

$$CI\% = \frac{CI}{EL} \times 100\% = \frac{0.897}{20.0} \times 100\% = 4.49\%$$

The resulting value for CI% is 4.49 percent. Because this value is less than 10 percent, the correlation satisfies the criterion for confidence interval half range.

How do I calculate the tolerance interval half range percentage for the linear correlation?

Equation 11-11 of PS-11 is used to calculated the tolerance interval half range (*TI*) at the mean value of x:

$$TI = k_T \cdot S_L$$

The parameter k_T is determined using Equation 11-12 of PS-11:

$$k_T = u_{n'} \cdot v_{df}$$

The parameters u_{nN} and v_{df} are determined from Table 1 of PS-11. In this example, nN is the number of test runs used for the correlation (20), u_{nN} = 1.177, and v_{df} equals 1.384 for n-2 degrees of freedom. With these values, k_T and TI are calculated as follows:

$$k_T = u_{n'} \cdot v_{df} = 1.177(1.384) = 1.629$$

$$TI = k_T \cdot S_L = 1.629(1.91) = 3.111$$

With an emission limit of 20.0 mg/acm, the tolerance interval as a percentage of the emission limit is determined using Equation 11-13 as follows:

$$TI\% = \frac{TI}{EL} \times 100\% = \frac{3.111}{20.0} \times 100\% = 15.6\%$$

The resulting value for TI% is 15.6 percent. Because this value is less than 25 percent, the correlation satisfies the criterion for tolerance interval half range. In summary, the linear correlation developed from the data satisfies all of the criteria specified in PS-11.

<u>How do I develop a polynomial correlation curve from the data?</u>

A polynomial correlation curve can be expressed by the following:

$$\hat{y} = b_0 + b_1 x + b_2 x^2$$

The procedures for developing a polynomial correlation curve are described in Section 12.3(2) of PS-11. The first step is to determine the values for the parameters S_1 through S_7, as defined in Equations 11-17 and 11-18 of PS-11. These calculations are summarized as follows:

$$S_1 = \sum_{i=1}^{n} (x_i) = (4.9 + 7.6 + 7.8 + \ldots + 33.1) = 370.7$$

$$S_2 = \sum_{i=1}^{n} (x_i^2) = (4.9^2 + 7.6^2 + 7.8^2 + \ldots + 33.1^2) = 8{,}312$$

$$S_3 = \sum_{i=1}^{n} (x_i^3) = (4.9^3 + 7.6^3 + 7.8^3 + \ldots + 33.1^3) = 208{,}377$$

$$S_4 = \sum_{i=1}^{n} (x_i^4) = (4.9^4 + 7.6^4 + 7.8^4 + \ldots + 33.1^4) = 5{,}574{,}977$$

$$S_5 = \sum_{i=1}^{n} (y_i) = (2.1 + 2.9 + 5.1 + \ldots + 18.5) = 189.4$$

$$S_6 = \sum_{i=1}^{n} (x_i y_i) = ((4.9)(2.1) + (7.6)(2.9) + (7.8)(5.1) + \ldots + (33.1)(18.5)) = 4{,}295$$

$$S_7 = \sum_{i=1}^{n} (x_i^2 y_i) = ((4.9^2)(2.1) + (7.6^2)(2.0) + (7.8^2)(5.1) + \ldots + (33.1^2)(18.5)) = 108{,}567$$

Next, the parameter det A must be calculated using Equation 11-22 of PS-11. For this example, the calculation is as follows:

$$\det A = n \cdot S_2 \cdot S_4 - S_2 \cdot S_2 \cdot S_2 + S_1 \cdot S_3 \cdot S_2 - S_3 \cdot S_3 \cdot n + S_2 \cdot S_1 \cdot S_3 - S_4 \cdot S_1 \cdot S_1 =$$
$$(20)(8{,}312)(5{,}574{,}977) - (8{,}312)(8{,}312)(8{,}312) + 370.7(208{,}377)(8{,}312) - (208{,}377)(208{,}377)(20) +$$
$$(8{,}312)(370.7)(208{,}377) - (5{,}574{,}977)(370.7)(370.7) = 2.120 \times 10^9$$

The values for the correlation curve parameters b_0, b_1, and b_2 can now be determined using Equations 11-19 to 11-21 of PS-11:

$$b_0 = \frac{\left(S_5 \square S_2 \square S_4 + S_1 \square S_3 \square S_7 + S_2 \square S_6 \square S_3 - S_7 \square S_2 \square S_2 - S_3 \square S_3 \square S_5 - S_4 \square S_6 \square S_1\right)}{\det A}$$

$$= \frac{1}{2.12 \times 10^9}[(189)(8{,}312)(5{,}574{,}977) + (3707)(208{,}377)(108{,}567) + (208{,}377)(4{,}295)(208{,}377) -$$
$$(108{,}567)(8{,}312)(8{,}312) - (208{,}377)(208{,}377)(189) - (5{,}574{,}977)(4{,}295)(3707)]$$

$$= 0.420$$

$$b_1 = \frac{\left(n \square S_6 \square S_4 + S_5 \square S_3 \square S_2 + S_2 \square S_1 \square S_7 - S_2 \square S_6 \square S_2 - S_7 \square S_3 \square n - S_4 \square S_1 \square S_5\right)}{det\ A}$$

$$= \frac{1}{2.12 \times 10^9}[(20)(4{,}295)(5{,}574{,}977) + (189)(208{,}377)(8{,}312) + (8{,}312)(3707)(108{,}567) -$$
$$(8{,}312)(4{,}295)(8{,}312) - (108{,}567)(208{,}377)(20) - (5{,}574{,}977)(3707)(189)]$$

$$= 0.406$$

$$b_2 = \frac{\left(n \square S_2 \square S_7 + S_1 \square S_6 \square S_2 + S_5 \square S_1 \square S_3 - S_2 \square S_2 \square S_5 - S_3 \square S_6 \square n - S_7 \square S_1 \square S_1\right)}{\det A}$$

$$= \frac{1}{2.12 \times 10^9}[(20)(8{,}312)(108{,}567) + (3707)(4{,}295)(8{,}312) + (189)(3707)(208{,}377) -$$
$$(8{,}312)(8{,}312)(189) - (208{,}377)(4{,}295)(20) - (108{,}567)(3707)(3707)]$$

$$= 0.00368$$

The resulting correlation curve is as follows:

$$\hat{y} = 0.420 + 0.406x + 0.00368x^2$$

<u>How do I determine the polynomial correlation coefficient?</u>

The polynomial correlation coefficient (r) is calculated using the parameters S_P and S_y. The parameter S_P is defined in Equation 11-26 as follows:

$$S_p = \sqrt{\frac{1}{n-3}\sum_{i=1}^{n}\left(\hat{y}_i - y_i\right)^2}$$

To calculate S_P, the predicted PM concentrations must be determined by substituting the PM CEMS response values into the correlation curve. For example, the predicted PM concentration for Run 10 is

$$\hat{y} = 0.420 + 0.406(12) + 0.00368(12)^2 = 5.819$$

Table Y-3 shows the predicted PM concentrations for all of the 20 test runs used to develop the correlation. Substitute these values into Equation 11-26 to calculate the value for S_p:

$$S_p = \sqrt{\frac{1}{20-3}\left((2.497-2.1)^2 + (3.717-2.9)^2 + (3.809-5.1)^2 + \ldots + (17.884-18.5)^2\right)} = 1.951$$

Table Y-3. Predicted PM Concentrations ($\hat{y}$) for Polynomial Correlation

Run no.	PM CEMS response, mA	PM concentration, mg/acm	Predicted PM concentration ($\hat{y}$)[a]
1	4.9	2.1	2.497
2	7.6	2.9	3.717
3	7.8	5.1	3.809
4	9.2	4.8	4.465
5	10.1	2.9	4.894
6	11.06	6.3	5.358
10	12	8.4	5.819
11	18.2	8.4	9.024
12	15.7	4.9	7.698
13	13.9	6.3	6.771
14	19.7	13	9.842
15	21.02	9.1	10.576
16	22.3	13.9	11.299
17	24.1	8.7	12.337
19	24.7	12.6	12.688
20	26.4	15.9	13.698
21	29.08	16.2	15.333
22	29.3	13.5	15.469
23	30.5	15.9	16.221
24	33.1	18.5	17.884

[a] Calculated using the correlation equation: $\hat{y} = 0.420 + 0.406x + 0.00368x^2$.

The parameter S_y was determined previously to be 5.095. As shown below, substituting S_y and S_P into Equation 11-33 results in a correlation coefficient of 0.924, which satisfies the minimum value of 0.85 specified by PS-11.

$$r = \sqrt{1 - \frac{S_p^{\,2}}{S_y^{\,2}}} = \sqrt{1 - \frac{(1.951)^2}{(5.095)^2}} = 0.924$$

<u>How do I calculate the confidence interval half range percentage for the polynomial correlation?</u>

The half range for the 95 percent CI for the predicted PM concentration must be determined at the minimum value of the parameter delta ( ) $_{min}$). The expression for calculating) is provided in Equation 11-25 of PS-11:

$$\Delta = C_0 + 2C_1 x + \left(2C_2 + C_3\right)x^2 + 2C_4 x^3 + C_5 x^4$$

Before calculating), it is first necessary to calculate the parameters C_0 to C_5, which are defined in Equation 11-23 of PS-11 as follows:

$$C_0 = \frac{\left(S_2 \square S_4 - S_3^{\ 2}\right)}{D}$$

$$C_1 = \frac{\left(S_3 \square S_2 - S_1 \square S_4\right)}{D}$$

$$C_2 = \frac{S_1 \square S_3 - S_2^{\ 2}}{D}$$

$$C_3 = \frac{\left(nS_4 - S_2^{\ 2}\right)}{D}$$

$$C_4 = \frac{\left(S_1 \square S_2 - nS_3\right)}{D}$$

$$C_5 = \frac{\left(nS_2 - S_1^{\ 2}\right)}{D}$$

The variable D in the denominator of these expressions is determined using Equation 11-24. For this example, this calculation is as follows:

$$D = n\left(S_2 \square S_4 - S_3^{\ 2}\right) + S_1\left(S_3 \square S_2 - S_1 \square S_4\right) + S_2\left(S_1 \square S_3 - S_2^{\ 2}\right)$$
$$= 20\left(\left(8{,}312\right)\left(5{,}574{,}977\right) - \left(208{,}377\right)^2\right) + 3707\left(\left(208{,}377\right)\left(8{,}312\right) - \left(3707\right)\left(5{,}574{,}577\right)\right) +$$
$$8{,}312\left(\left(3707\right)\left(208{,}377\right) - \left(8{,}312\right)^2\right) = 2.120 \times 10^9$$

Substituting the values for D and for the variables S_1 to S_4, which were defined previously, you find the following coefficients for determining):

$$C_0 \frac{\left(\left(8{,}312\right)\left(5{,}574{,}977\right) - \left(208{,}377\right)^3\right)}{2.12 \times 10^9} = 1.375$$

$$C_1 = \frac{\left((208{,}377)(8{,}312) - (3707)(5{,}574{,}977)\right)}{2.12 \times 10^9} = -0.1578$$

$$C_2 = \frac{\left((3707)(208{,}377) - (8{,}312)^2\right)}{2.12 \times 10^9} = 0.003846$$

$$C_3 = \frac{\left((20)(5{,}574{,}977) - (8{,}312)^2\right)}{2.12 \times 10^9} = 0.02001$$

$$C_4 = \frac{\left((3707)(8{,}312) - (20)(208{,}377)\right)}{2.12 \times 10^9} = -0.00005126$$

$$C_5 = \frac{\left((20)(8{,}312) - (3707)^2\right)}{2.12 \times 10^9} = 1.360 \times 10^{-5}$$

When these values are substituted, Equation 11-25 becomes the following:

$$\Delta = 1.375 + 2(-0.1578)x + \left((2)(0.003846) + 0.02001\right)x^2 + 2(-0.00005126)x^3 + 1.36 \times 10^{-5} x^4$$
$$= 1.375 - 0.3155\,x + 0.0277\,x^2 - 0.001025\,x^3 + 1.36 \times 10^{-5} x^4$$

Table Y-4 summarizes the results when the PM CEMS response values for each test run are substituted into the above equation.

Table Y-4. Values for)

Run no.	PM CEMS response, mA	PM concentration, mg/acm	x^2	x^3	x^4	)
1	4.9	2.1	24.01	118	576	0.3817
2	7.6	2.9	57.76	439	3,336	0.1727
3	7.8	5.1	60.84	475	3,702	0.1634
4	9.2	4.8	84.64	779	7,164	0.1162
5	10.1	2.9	102.01	1,030	10,406	0.0995
6	11.06	6.3	122.32	1,353	14,963	0.0906
10	12.0	8.4	144.00	1,728	20,736	0.0884
11	18.2	8.4	331.24	6,029	109,720	0.1202
12	15.7	4.9	246.49	3,870	60,757	0.1084
13	13.9	6.3	193.21	2,686	37,330	0.0960
14	19.7	13.0	388.09	7,645	150,614	0.1204
15	21.02	9.1	441.84	9,287	195,223	0.1162
16	22.3	13.9	497.29	11,090	247,297	0.1092

(continued)

Table Y-4. (continued)

Run no.	PM CEMS response, mA	PM concentration, mg/acm	x^2	x^3	x^4	)
17	24.1	8.7	580.81	13,998	337,340	0.0985
19	24.7	12.6	610.09	15,069	372,210	0.0959
20	26.4	15.9	696.96	18,400	485,753	0.0960
21	29.08	16.2	845.65	24,591	715,118	0.1417
22	29.3	13.5	858.49	25,154	737,005	0.1492
23	30.5	15.9	930.25	28,373	865,365	0.2046
24	33.1	18.5	1,095.61	36,265	1,200,361	0.4312

The minimum value for) is 0.0884 and corresponds to Run 10. Using this value for $)_{min}$, the value for S_P calculated previously, and the *t* statistic with n-3 degrees of freedom, you can calculate the confidence interval half range using Equation 11-27.

$$CI = t_{df} \square S_p \sqrt{\Delta_{\min}} = (2.11)(1.951)\sqrt{0.0884} = 1.223$$

The confidence interval half range as a percentage of the emission limit is calculated using Equation 11-28 as follows:

$$CI\% = \frac{CI}{EL} \times 100\% = \frac{1.224}{20} \times 100\% = 6.12\%$$

The resulting value for CI% is 6.12 percent. Because this value is less than10 percent, the correlation satisfies the criterion for confidence interval half range.

<u>How do I determine the tolerance interval half range percentage for the polynomial correlation?</u>

To calculate the tolerance interval half range, the parameter nNmust be calculated using Equation 11-31 of PS-11.

$$n' = \frac{1}{\Delta_{\min}} = \frac{1}{0.0884} = 11.31$$

With nNrounded to the nearest integer (11), the values for the variables u_{nN} and v_{df} with n-3 degrees of freedom are taken from Table 1 of PS-11 and substituted into Equation 11-30 to determine k_T:

$$k_T = u_{n'} \square v_{df} = (1.195)(1.400) = 1.673$$

Using this value for k_T and the value for S_P calculated previously, the tolerance interval half range is calculated using Equation 11-29 as follows:

$$TI = k_T \cdot S_P = (1.673)(1.951) = 3.263$$

Finally, to determine the tolerance interval as a percentage of the emission limit (TI%), Equation 11-32 is used as follows:

$$TI\% = \frac{TI}{EL} \times 100\% = \frac{3.263}{20.0} \times 100\% = 16.3\%$$

The resulting value for TI% is 16.3 percent. Because this value is less than 25 percent, the correlation satisfies the criterion for tolerance interval half range. In summary, the polynomial correlation developed from the data satisfies all of the criteria specified in PS-11.

<u>How do I develop a logarithmic correlation curve from the data?</u>

A logarithmic correlation curve is expressed as follows:

$$\hat{y} = b_0 + b_1 ln(x) = b_0 + b_1 x'$$

Developing a logarithmic correlation entails following the same procedures used to develop a linear correlation. However, instead of using the PM CEMS response (x) values, the natural logarithm of the PM CEMS values (x' values) are used in the calculations. Table Y-5 summarizes the test data, including the corresponding values of x'.

Table Y-5. Data for Developing Logarithmic Correlation

Run no.	PM CEMS response, mA	Logarithm of PM CEMS response x' = ln (x)	PM concentration, mg/acm
1	4.9	1.589	2.1
2	7.6	2.028	2.9
3	7.8	2.054	5.1
4	9.2	2.219	4.8
5	10.1	2.313	2.9
6	11.06	2.403	6.3
10	12	2.485	8.4
11	18.2	2.901	8.4
12	15.7	2.754	4.9
13	13.9	2.632	6.3
14	19.7	2.981	13
15	21.02	3.045	9.1

(continued)

Table Y-5. (continued)

Run no.	PM CEMS response, mA	Logarithm of PM CEMS response x' = ln (x)	PM concentration, mg/acm
16	22.3	3.105	13.9
17	24.1	3.182	8.7
19	24.7	3.207	12.6
20	26.4	3.273	15.9
21	29.08	3.370	16.2
22	29.3	3.378	13.5
23	30.5	3.418	15.9
24	33.1	3.500	18.5

Following the same procedures described previously for developing a linear correlation curve, you can calculate values for the parameters $\bar{x}'$ and $\bar{y}$, using Equation 11-5; $S_{x'x'}$ and $S_{x'y'}$, using Equation 11-7; b_1, using Equation 11-6; and b_0, using Equation 11-4:

$$\bar{x}' = \frac{1}{20}(1.589 + 2.028 + 2.054 + \ldots + 3.500) = 2.79$$

$$\bar{y} = \frac{1}{20}(21 + 29 + 51 + \ldots + 185) = 94.7$$

$$S_{x'x'} = (1.589 - 2.79)^2 + (2.028 - 2.79)^2 + (2.054 - 2.79)^2 + \ldots + (3.50 - 2.79)^2 = 5.74$$

$$S_{x'y'} = (1.589 - 2.79)(21 - 94.7) + (2.028 - 2.79)(29 - 94.7) + \ldots + (3.50 - 2.79)(185 - 94.7) = 478$$

$$b_1 = \frac{478}{5.74} = 83.23$$

$$b_0 = 94.7 - (83.23)(2.79) = -137.7$$

The resulting equation for the logarithmic correlation curve is

$$\hat{y} = -138 + 832 \cdot ln(x)$$

<u>How do I determine the logarithmic correlation coefficient?</u>

Determining the logarithmic correlation coefficient requires calculating the predicted PM concentration ($\hat{y}$) for each test run, the parameter S_L, and the parameter S_y following the same procedure used to calculate the linear correlation coefficient. The only difference is that the predicted PM concentrations are calculated using the logarithmic correlation equation. Table Y-6 shows the PM concentrations calculated using the logarithmic correlation coefficient.

Table Y-6. Predicted PM Concentrations for the Logarithmic Correlation

Run no.	PM CEMS response, mA	Logarithm of PM CEMS response x' = ln (x)	PM concentration, mg/acm	Predicted PM concentration (í)[a]
1	4.9	1.589	2.1	-0.539
2	7.6	2.028	2.9	3.114
3	7.8	2.054	5.1	3.330
4	9.2	2.219	4.8	4.704
5	10.1	2.313	2.9	5.481
6	11.06	2.403	6.3	6.237
10	12	2.485	8.4	6.916
11	18.2	2.901	8.4	10.382
12	15.7	2.754	4.9	9.152
13	13.9	2.632	6.3	8.139
14	19.7	2.981	13	11.041
15	21.02	3.045	9.1	11.581
16	22.3	3.105	13.9	12.073
17	24.1	3.182	8.7	12.719
19	24.7	3.207	12.6	12.924
20	26.4	3.273	15.9	13.478
21	29.08	3.370	16.2	14.283
22	29.3	3.378	13.5	14.345
23	30.5	3.418	15.9	14.679
24	33.1	3.500	18.5	15.360

[a] Calculated using the logarithmic correlation equation: í = ! 13.8 + 8.32@*ln*(*x*).

With the values for í shown in Table Y-6, the calculation for S_L is as follows:

$$S_L = \sqrt{\frac{1}{(20-2)}\left((-0539-21)^2 + (3.114-29)^2 + K + (1536-185)^2\right)} = 2305$$

The value for S_y is the same as was calculated for the linear correlation (5.095). Substituting the values for S_L and S_y into Equation 11-14, the logarithmic correlation coefficient is

$$r = \sqrt{1 - \frac{(2.305)^2}{(5.095)^2}} = 0892$$

The logarithmic correlation coefficient (0.892) is greater than the minimum value specified by PS-11 (0.85) and is acceptable.

How do I determine the confidence interval half range percentage for the logarithmic correlation?

The confidence interval half range for the logarithmic correlation is determined using the same procedures used for the linear correlation. Substituting the value for S_L calculated above (2.305) and the same value for the *t*-statistic (2.101) used for the linear correlation, Equation 11-8 yields

$$CI = (2.101)(2.305)\sqrt{\frac{1}{20}} = 1.083$$

The emission limit in this example is 20.0 mg/acm. To determine CI as a percentage of the emission limit, Equation 11-10 is used as follows:

$$\frac{1.083}{20} \times 100\% = 5.41\%$$

The resulting value for CI% is 5.41 percent. Because this value is less than 10 percent, the correlation satisfies the criterion for confidence interval half range.

How do I determine the tolerance interval half range percentage for the logarithmic correlation?

The tolerance interval half range for the logarithmic correlation is determined using the same procedures used for the linear correlation. Substituting the value for S_L calculated above (2.305) and the same value for the k_T used for the linear correlation (1.629), Equation 11-11 yields

$$TI = (1.629)(2.305) = 3.75$$

The emission limit in this example is 20.0 mg/acm. To determine TI as a percentage of the emission limit, Equation 11-13 is used as follows:

$$\frac{3.75}{200} \times 100\% = 188\%$$

The resulting value for TI% is 18.8 percent. Because this value is less than 25 percent, the correlation satisfies the criterion for confidence interval half range.

How do I develop an exponential correlation curve from the data?

An exponential correlation curve is expressed as follows:

$$\hat{y} = b_0 e^{b_1 x}$$

To develop an exponential correlation curve, follow the same procedures used to develop a linear correlation curve. However, instead of using the PM CEMS concentrations (y) values, use

the natural logarithm of the PM CEMS concentrations (y' values) to perform the calculations. Table Y-7 summarizes the test data, including the corresponding values of y'.

Table Y-7. Transformed Data for Developing Exponential Correlation

Run no.	PM CEMS response, mA	PM concentration, mg/acm	Logarithm of PM CEMS response y' = ln (y)
1	4.9	2.1	0.742
2	7.6	2.9	1.065
3	7.8	5.1	1.629
4	9.2	4.8	1.569
5	10.1	2.9	1.065
6	11.06	6.3	1.841
10	12	8.4	2.128
11	18.2	8.4	2.128
12	15.7	4.9	1.589
13	13.9	6.3	1.841
14	19.7	13	2.565
15	21.02	9.1	2.208
16	22.3	13.9	2.632
17	24.1	8.7	2.163
19	24.7	12.6	2.534
20	26.4	15.9	2.766
21	29.08	16.2	2.785
22	29.3	13.5	2.603
23	30.5	15.9	2.766
24	33.1	18.5	2.918

Following the same procedures described previously for developing a linear correlation curve, calculate values for the parameters $\bar{x}$ and $\bar{y}'$ using Equation 11-5; S_{xx} and $S_{xy'}$ using Equation 11-7; b_1' using Equation 11-6; and b_0' using Equation 11-4:

$$\bar{x} = \frac{1}{20}\left(4.9 + 7.6 + 7.8 + \ldots + 33.1\right) = 18.53$$

$$\bar{y}' = \frac{1}{20}\left(0.742 + 1.065 + 1.629 + \ldots + 2.918\right) = 2.077$$

$$S_{xx} = \left((4.9 - 18.53)^2 + (7.6 - 18.53)^2 + \ldots + (33.1 - 18.53)^2\right) = 1,442.2$$

$$S_{xy'} = \left((4.9 - 18.53)(0.742 - 2.077) + (7.6 - 18.53)(1.065 - 2.077) + \ldots + (33.1 - 18.53)(2.918 - 2.077)\right) = 96.58$$

$$b_1 = \frac{9658}{14422} = 00670$$

$$b_0' = 2077 - (00670)(1853) = 08357$$

The resulting equation is expressed in the following form:

$$\$' = b_0' + b_1' x$$

For this example, the correlation equation becomes

$$\$' = 0.836 + 0.0670x$$

Converting this to the format that yields the predicted PM concentration in the correct units entails taking the exponential of each side of the equation, as follows:

$$Exp(\$') = Exp(08357 + 00670\,x)$$
$$\$ = 2.31\,e^{0.0670x}$$

In the resulting correlation equation, the coefficient 2.31 is simply the exponential of b_0.

<u>How do I determine the exponential correlation coefficient?</u>

Determining the exponential correlation coefficient requires calculating the predicted PM concentration ($\hat{y}$) for each test run, the parameter S_L, and the parameter S_{yN} following the same procedure used to calculate the linear correlation coefficient. Table Y-8 shows the PM concentrations calculated using the linear form (i.e., $\hat{y} = b_0 + b_1 x$) of the exponential correlation equation.

Table Y-8. Predicted PM Concentrations Using Exponential Correlation Equation

Run no.	PM CEMS response, mA	PM concentration, mg/acm	Logarithm of PM CEMS response y' = ln (y)	Predicted PM concentration ($\hat{y}$)[a]
1	4.9	2.1	0.742	1.164
2	7.6	2.9	1.065	1.345
3	7.8	5.1	1.629	1.358
4	9.2	4.8	1.569	1.452
5	10.1	2.9	1.065	1.512
6	11.06	6.3	1.841	1.576
10	12	8.4	2.128	1.639
11	18.2	8.4	2.128	2.055
12	15.7	4.9	1.589	1.887

(continued)

Table Y-8. (continued)

Run no.	PM CEMS response, mA	PM concentration, mg/acm	Logarithm of PM CEMS response y' = ln (y)	Predicted PM concentration (íN)[a]
13	13.9	6.3	1.841	1.767
14	19.7	13	2.565	2.155
15	21.02	9.1	2.208	2.243
16	22.3	13.9	2.632	2.329
17	24.1	8.7	2.163	2.450
19	24.7	12.6	2.534	2.490
20	26.4	15.9	2.766	2.604
21	29.08	16.2	2.785	2.783
22	29.3	13.5	2.603	2.798
23	30.5	15.9	2.766	2.878
24	33.1	18.5	2.918	3.052

[a] Calculated using the exponential correlation equation: íN= 0.836 + 0.067*x*.

Using the values for íNshown in Table Y-8, calculate S_L as follows:

$$S_L - \sqrt{\frac{1}{(20-2)}\left((1.164-0.742)^2+(1.345-1.065)^2+K+(3.052-2.918)^2\right)} = 0.278$$

The value for S_{yN} is determined using Equation 11-15 with the values for yNsubstituted for the values of y, as follows:

$$S_y = \sqrt{\frac{\left((0.742-2.077)^2+(1.065-2.077)^2+K+(2.918-2.077)^2\right)}{(20-1)}} = 0.643$$

Substitute the values for S_L and S_{yN} into Equation 11-14 to calculate the exponential correlation coefficient:

$$r = \sqrt{1-\frac{(0.278)^2}{(0.643)^2}} = 0.902$$

The exponential correlation coefficient (0.902) is greater than the minimum value specified by PS-11 (0.85) and is acceptable.

<u>How do I determine the confidence interval half range percentage for the exponential correlation?</u>

The confidence interval half range for the exponential correlation is determined using Equation 11-8, which is the same equation used to calculate the confidence interval half range

for linear correlations. Because the confidence interval half range is calculated on the log scale, it is designated as CI′ for the exponential model. Using Equation 11-8, CI′ is calculated as follows:

$$CI' = t_{df,\ 1-\alpha/2} \cdot S_L \sqrt{\frac{1}{n}} = (2.101)(0.278)\sqrt{\frac{1}{20}} = 0.1307$$

The lower confidence limit (LCL′) and upper confidence limit (UCL′) are calculated using Equations 11-40 and 11-41, respectively[1]

$$LCL' = \bar{y}' - CI' = 2.077 - 0.1307 = 1.946$$

$$UCL' = \bar{y}' + CI' = 2.077 + 0.1307 = 2.207$$

Next, the confidence interval half range (CI) must be calculated on the original PM concentration scale using Equation 11-42 from PS-11

$$CI = \frac{(e^{UCL'} - e^{LCL'})}{2} = \frac{(e^{2.207} - e^{1.946})}{2} = 1.045$$

The confidence interval as a percentage of the emission limit is calculated using Equation 11-10 as follows:

$$CI\% = \frac{1.045}{20} \times 100\% = 523\%$$

The resulting value for CI% is 5.23 percent. Because this value is less than 10 percent, the correlation satisfies the criterion for confidence interval half range.

<u>How do I determine the tolerance interval half range percentage for the exponential correlation?</u>

The tolerance interval half range for the exponential correlation also is calculated using the same procedure as is used for linear models. Because it is calculated on the log scale, it is denoted here as TI′. Using Equation 11-11, TI′ is calculated as follows:

$$TI' = k_T \cdot S_L = (1.629)(0.278) = 0.453$$

The lower tolerance limit (LTL′) and upper tolerance limit (UTL′) are calculated using Equations 11-43 and 11-44, respectively

$$LTL' = \bar{y}' - TI' = 2.077 - 0.453 = 1.624$$

$$UTL' = \bar{y}' + TI' = 2.077 + 0.453 = 2.530$$

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[1] The equations presented in this section for exponential and power models correspond to the amended version of PS-11, which was not yet published at the time this was prepared.

Next, the tolerance interval half range (TI) must be calculated on the original PM concentration scale using Equation 11-45 from PS-11

$$TI = \frac{(e^{UTL'} - e^{LTL'})}{2} = \frac{(e^{2.53} - e^{1.624})}{2} = 3.74$$

The tolerance interval as a percentage of the emission limit is calculated using Equation 11-13 as follows:

$$TI\% = \frac{3.74}{20} \times 100\% = 187\%$$

The resulting value for TI% is 18.7 percent. Because this value is less than 25 percent, the correlation satisfies the criterion for confidence interval half range. In summary, the exponential model satisfies the PS-11 criteria for correlation coefficient, confidence interval half range percentage, and tolerance interval half range percentage.

<u>How do I develop a power correlation curve from the data?</u>

A power correlation curve has the following form:

$$\hat{y} = b_0 x^{b_1}$$

Developing a power correlation requires transforming both the PM CEMS response (x) values, as described previously for a logarithmic correlation, and the PM concentration (y) values, as was described above for an exponential correlation. The procedures used to develop a linear correlation are then followed using the transformed values for x and y. Table Y-9 shows the test data and transformed values that must be used.

Table Y-9. Summary of Test Data and Transformed Values for Developing Power Correlation

Run no.	PM CEMS response, mA	Logarithm of PM CEMS response x' = ln (x)	PM concentration, mg/acm	Logarithm of PM CEMS response y' = ln (y)
1	4.9	1.589	2.1	0.742
2	7.6	2.028	2.9	1.065
3	7.8	2.054	5.1	1.629
4	9.2	2.219	4.8	1.569
5	10.1	2.313	2.9	1.065
6	11.06	2.403	6.3	1.841
10	12	2.485	8.4	2.128
11	18.2	2.901	8.4	2.128

(continued)

Table Y-9. (continued)

Run no.	PM CEMS response, mA	Logarithm of PM CEMS response x' = ln (x)	PM concentration, mg/acm	Logarithm of PM CEMS response y' = ln (y)
12	15.7	2.754	4.9	1.589
13	13.9	2.632	6.3	1.841
14	19.7	2.981	13	2.565
15	21.02	3.045	9.1	2.208
16	22.3	3.105	13.9	2.632
17	24.1	3.182	8.7	2.163
19	24.7	3.207	12.6	2.534
20	26.4	3.273	15.9	2.766
21	29.08	3.370	16.2	2.785
22	29.3	3.378	13.5	2.603
23	30.5	3.418	15.9	2.766
24	33.1	3.500	18.5	2.918

Following the same procedures described previously for developing a linear correlation curve, calculate values for the parameters $\bar{x}'$ and $\bar{y}'$ using Equation 11-5; $S_{x'x'}$ and $S_{x'y'}$ using Equation 11-7; b_1' using Equation 11-6; and b_0' using Equation 11-4:

$$\bar{x}' = \frac{1}{20}(1.589 + 2.028 + 2.054 + \ldots + 3.5) = 2.792$$

$$\bar{y}' = \frac{1}{20}(0.742 + 1.065 + 1.629 + \ldots + 2.918) = 2.077$$

$$S_{x'x'} = \left((1.589 - 2.792)^2 + (2.028 - 2.792)^2 + \ldots + (3.50 - 2.792)^2\right) = 5.74$$

$$S_{x'y'} = (1.588 - 2.792)(0.742 - 2.077) + (2.028 - 2.792)(1.065 - 2.077) + \ldots + (3.50 - 2.792)(2.918 - 2.077) = 6.22$$

$$b_1 = \frac{6.22}{5.74} = 1.0839$$

$$b_0' = 2.077 - (1.0839)(2.792) = -0.9492$$

The resulting equation is expressed in the following form:

$$\hat{y}' = b_0' + b_1' x'$$

For this example, the correlation equation becomes

$$\hat{y}' = -0.9492 + 1.084\, x'$$

Converting this to the format that yields the predicted PM concentration in the correct units entails taking the exponential of each side of the equation, as follows:

$$Exp(\text{íN}) = Exp(-0.9492 + 1.084x\text{N})$$

$$\text{í} = 0.387x^{1.084}$$

In this equation, the coefficient 0.387 is the exponential of -0.9492, the value for b_0N

<u>How do I determine the power correlation coefficient?</u>

To determine the power correlation coefficient, the predicted PM concentration (íN) must be calculated for each test run. Table Y-10 shows the predicted PM concentrations calculated using the linear form (i.e., íN= b_0N+ b_1NxN) of the power correlation equation.

Table Y-10. Predicted PM Concentrations Using the Power Correlation Equation

Run no.	PM CEMS response, mA	Logarithm of PM CEMS response x' = ln (x)	PM concentration, mg/acm	Logarithm of PM CEMS response y' = ln (y)	Predicted PM concentration (íN)[a]
1	4.9	1.589	2.1	0.742	0.773
2	7.6	2.028	2.9	1.065	1.249
3	7.8	2.054	5.1	1.629	1.277
4	9.2	2.219	4.8	1.569	1.456
5	10.1	2.313	2.9	1.065	1.557
6	11.06	2.403	6.3	1.841	1.656
10	12	2.485	8.4	2.128	1.744
11	18.2	2.901	8.4	2.128	2.196
12	15.7	2.754	4.9	1.589	2.035
13	13.9	2.632	6.3	1.841	1.903
14	19.7	2.981	13	2.565	2.281
15	21.02	3.045	9.1	2.208	2.352
16	22.3	3.105	13.9	2.632	2.416
17	24.1	3.182	8.7	2.163	2.500
19	24.7	3.207	12.6	2.534	2.527
20	26.4	3.273	15.9	2.766	2.599
21	29.08	3.370	16.2	2.785	2.704
22	29.3	3.378	13.5	2.603	2.712
23	30.5	3.418	15.9	2.766	2.755
24	33.1	3.500	18.5	2.918	2.844

[a] Calculated using the exponential correlation equation: íN= !0.9492 + 1.084xN

The parameter S_L is calculated using Equation 11-9 with the predicted values ($\hat{y}'$) given in the above table, and the parameter $S_{y'}$ is calculated following the same procedure used for the exponential correlation coefficient, as follows:

$$S_L = \sqrt{\frac{1}{(20-2)}\left((0.773-0.742)^2+(1.249-1.065)^2+\ldots+(2.844-2.918)^2\right)} = 0.249$$

$$S_{y'} = \sqrt{\frac{\left((0.742-2.077)^2+(1.065-2.077)^2+\ldots+(2.918-2.077)^2\right)}{20-1}} = 0.643$$

Substituting the values for S_L and $S_{y'}$ into Equation 11-14, the power correlation coefficient is

$$r = \sqrt{1-\frac{(0.249)^2}{(0.643)^2}} = 0.922$$

The correlation coefficient for the power model (0.922) is greater than the minimum value specified by PS-11 (0.85) and is acceptable.

<u>How do I determine the confidence interval half range percentage for the power correlation?</u>

The confidence interval half range for the power correlation is determined using the same procedures used for the exponential correlation. Because the confidence interval half range is calculated on the log scale, it is designated as CI' for the exponential model. Using Equation 11-8, CI' is calculated as follows:

$$CI' = (2.101)(0.249)\sqrt{\frac{1}{20}} = 0.117$$

The lower confidence limit (LCL') and upper confidence limit (UCL') are calculated using Equations 11-40 and 11-41, respectively

$$LCL' = \hat{y}' - CI' = 2.077 - 0.117 = 1.96$$

$$UCL' = \hat{y}' + CI' = 2.077 + 0.117 = 2.194$$

Next, the confidence interval half range (CI) must be calculated on the original PM concentration scale using Equation 11-42 from PS-11

$$CI = \frac{(e^{UCL'} - e^{LCL'})}{2} = \frac{(e^{2.194} - e^{1.96})}{2} = 0.936$$

The confidence interval as a percentage of the emission limit is calculated using Equation 11-10 as follows:

$$CI\% = \frac{0936}{20} \times 100\% = 468\%$$

The resulting value for CI% is 4.68 percent. Because this value is less than 10 percent, the correlation satisfies the criterion for confidence interval half range.

How do I determine the tolerance interval half range percentage for the power correlation?

The tolerance interval half range for the power correlation also is calculated using the same procedure used for the exponential model. Because it is calculated on the log scale, it is denoted here as TI_N. Using Equation 11-11, TI_N is calculated as follows:

$$TI_N = k_{TI} \cdot S_L = (1.629)(0.249) = 0.406$$

The lower tolerance limit (LTL_N) and upper tolerance limit (UTL_N) are calculated using Equations 11-43 and 11-44, respectively

$$LTL_N = \bar{y}_N - TI_N = 2.077 - 0.406 = 1.671$$

$$UTL_N = \bar{y}_N + TI_N = 2.077 + 0.406 = 2.483$$

Next, the tolerance interval half range (TI) must be calculated on the original PM concentration scale using Equation 11-45 from PS-11

$$TI = \frac{(e^{UTL'} - e^{LTL'})}{2} = \frac{(e^{2.483} - e^{1671})}{2} = 3.327$$

The tolerance interval as a percentage of the emission limit is calculated using Equation 11-13 as follows:

$$TI\% = \frac{3327}{20} \times 100\% = 166\%$$

The resulting value for TI% is 16.6 percent. Because this value is less than 25 percent, the correlation satisfies the criterion for confidence interval half range. In summary, the power model satisfies the PS-11 criteria for correlation coefficient, confidence interval half range percentage, and tolerance interval half range percentage.

Which model is the best fit for the data?

Table Y-11 summarizes the results of the analysis for all of the correlation models. As indicated in the table, all five models met the acceptance criteria specified in Section 13.2 of PS-11. The linear model had the highest correlation coefficient and would be considered the best fit for the data. However, any of the models could be used to show compliance. The complete output for the spreadsheet for this example is provided in Appendix A.1.

Table Y-11. Summary of Results for Facility A Correlation Test

Model type	Correlation equation	Correlation coefficient (r)	Confidence interval half range	Tolerance interval half range
Acceptance Criteria for PS-11		$ 0.85	# 10%	# 25%
Linear	í = -0.621 + 0.544x	0.927	4.49%	15.6%
Polynomial	í = 0.420 + 0.406x + 0.00368 x^2	0.924	6.12%	16.3%
Logarithmic	í = -13.8 + 8.32Ln(x)	0.892	5.41%	18.8%
Exponential	í = $2.31e^{0.0670x}$	0.902	5.23%	18.7%
Power	í = $0.387x^{1.08}$	0.922	4.68%	16.6%

Y.2 EXAMPLE 2: HOW DO I DEVELOP A CORRELATION WITH PAIRED SAMPLING TRAIN DATA?

Y.2.1 Correlation Test Scenario

A correlation test is performed on an incinerator at Facility B. The incinerator is equipped with an in situ PM CEMS that responds to PM concentrations at actual conditions. The emission limit, when converted to units that are consistent with the PM CEMS sampling conditions, is 24.3 mg/acm. Fifteen test runs are performed using Method 5i with paired sampling trains (Train A and Train B). No problems were reported for the test. Table Y-12 summarizes the correlation test data. Figure Y-2 shows a plot of the data.

Table Y-12. Summary of Correlation Test Data for Facility B

Run no.	PM CEMS response, mA	PM concentration, mg/acm	
		Sampling Train A	Sampling Train B
1	6.4	1.3	1.2
2	7.5	2.3	2.5
3	11.2	2.5	2.8
4	16.4	7.7	7.9
5	29.9	12.9	12.5
6	22.2	7.8	7.3
7	19.5	5.8	5.9
8	27.4	11.7	12.0
9	32.8	20.4	20.9
10	26.5	20.9	21.3
11	33.5	22.3	22.6
12	28.1	18.4	17.58
13	9.6	3.7	3.9
14	5.3	4.2	4.3
15	18.1	6.8	7.1

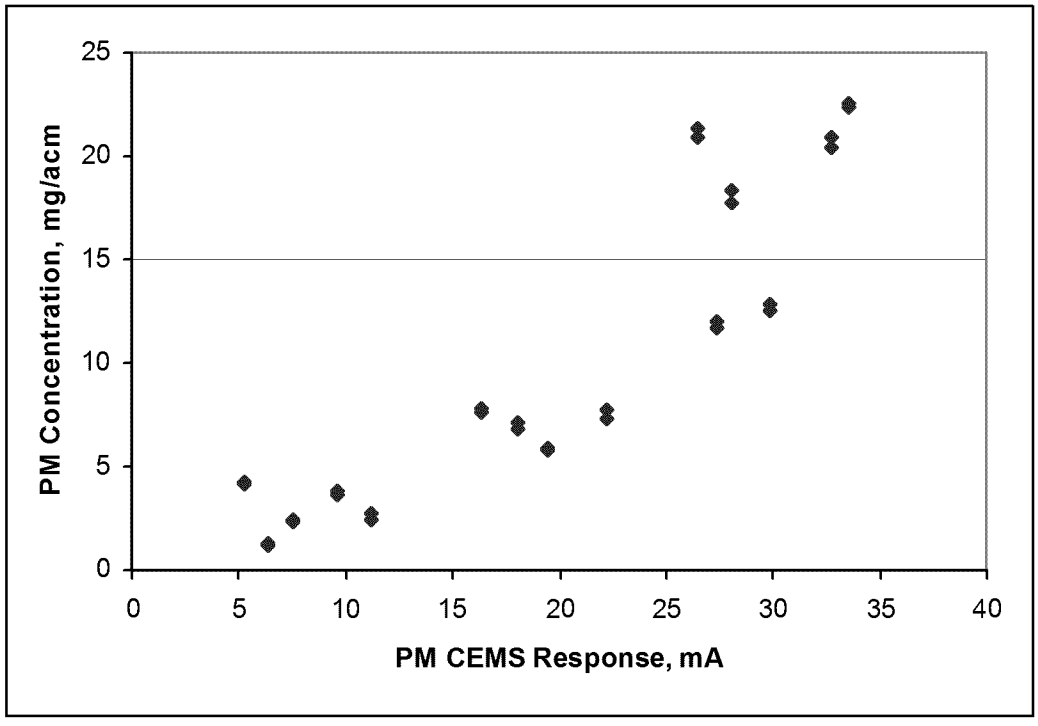


Figure Y-2. Plot of correlation test data for Facility B.

Should I average the two PM concentrations for each test run?

You should not average the two PM concentrations for any of the test runs. Instead, you should treat each PM concentration as a separate data point, as specified in Section 12.3 of PS-11. This means that for each run, the data set includes two PM concentrations and one PM CEMS response value. In other words, the correlation should be developed as if there were 30 test runs with one PM concentration for each run. Table Y-13 shows the data in the form that should be used for developing the correlation.

Before proceeding with the correlation, paired sampling train data should be checked for precision and bias. Guidance on precision and bias is provided separately. For this example, it is assumed that the data meet the criteria of precision and bias and are acceptable.

Table Y-13. Paired Sampling Train Test Data in Format for Developing Correlation

Run No.	PM CEMS Response, mA	PM Concentration, mg/acm
1A	6.4	1.3
1B	6.4	1.2
2A	7.5	2.3
2B	7.5	2.5
3A	11.2	2.5
3B	11.2	2.8
4A	16.4	7.7
4B	16.4	7.9
5A	29.9	12.9
5B	29.9	12.5
6A	22.2	7.8
6B	22.2	7.3
7A	19.5	5.8
7B	19.5	5.9
8A	27.4	11.7
8B	27.4	12.0
9A	32.8	20.4
9B	32.8	20.9
10A	26.5	20.9
10B	26.5	21.3
11A	33.5	22.3
11B	33.5	22.6
12A	28.1	18.4
12B	28.1	17.8
13A	9.6	3.7
13B	9.6	3.9
14A	5.3	4.2
14B	5.3	4.3
15A	18.1	6.8
15B	18.1	7.1

What are the results of the correlation analysis?

Table Y-14 summarizes the results of the analysis. The complete output for the spreadsheet for this example is provided in Appendix A.2.

Table Y-14. Summary of Results for Facility B Correlation Test

Model type	Correlation equation	Correlation coefficient (r)	Confidence interval half range	Tolerance interval half range
Acceptance Criteria for PS-11		$ 0.85	# 10%	# 25%
Linear	í = -3.24 + 0.672*x*	0.895	4.98%	20.0%
Polynomial	í = 2.85 - 0.173*x* + 0.0220 x^2	0.919	5.91%	18.0%
Logarithmic	í = -17.7 + 9.81Ln(*x*)	0.822	6.37%	25.6%
Exponential	í = $1.43e^{0.0828x}$	0.921	3.80%	15.9%
Power	í = $0.201x^{1.27}$	0.892	4.41%	18.7%

Which model should I use?

All of the models except the logarithmic model satisfy the acceptance criteria for PS-11. The logarithmic model fails to meet the minimum allowable correlation coefficient of 0.85. In addition, the tolerance interval half range percentage for the logarithmic model exceeds the allowable percentage of 25 percent. Based on the correlation coefficient, the exponential model is the best fit. However, the linear, polynomial, and power models could also be used.

Y.3 EXAMPLE 3: HOW DO I DEVELOP THE INITIAL CORRELATION?

Y.3.1 Correlation Test Scenario

A correlation test is performed on a hazardous waste incinerator at Facility C. The incinerator is equipped with an extractive PM CEMS that extracts and heats the samples to 121°C (250°F). The incinerator is subject to 40 CFR 63, subpart EEE, and must meet an emission limit of 34.0 mg/dscm corrected to 7 percent oxygen. Fifteen test runs are performed using a Method 5 sampling train. The average stack gas pressure, moisture content, and oxygen content during the test were 760 millimeters of mercury, 12 percent moisture, and 10 percent oxygen, respectively. No problems were reported for the test, and all data points will be used for the correlation. Table Y-15 summarizes the correlation test data. Figure Y-3 shows a plot of the data.

Table Y-15. Summary of Correlation Test Data for Facility C

Run no.	PM CEMS response, mA	PM concentration, mg/acm
1	5.8	3.5
2	7.6	9.0
3	7.8	7.8
4	16.4	25.2
5	10.1	16.6
6	11.1	14.3
7	12.0	19.6
8	13.2	20.1
9	15.7	23.4
10	17.6	26.7
11	21.7	26.8
12	21.0	28.9
13	22.3	25.7
14	24.1	26.2
15	24.7	26.2

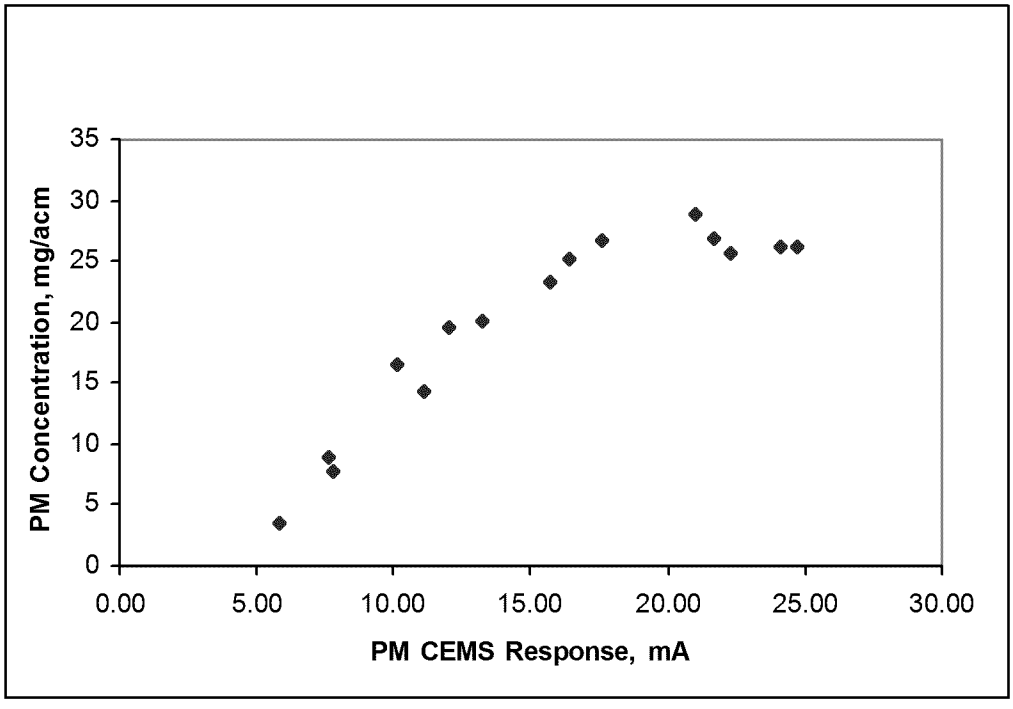


Figure Y-3. Plot of correlation test data for Facility C.

Do I need to convert the emission limit to other units for determining compliance with PS-11?

As indicated in Section 13.2 of PS-11, you must convert the emission limit to units that are consistent with the measurement conditions of your PM CEMS. For this example, you must convert the emission limit, which is specified in units of mg/dscm at 7 percent oxygen, to match the conditions under which the PM CEMS is operating. The following calculations show this conversion:

$$C_{S@7\%O_2} = C_a \times \frac{T_a}{T_{std}} \times \frac{P_{std}}{P_a} \times \frac{1}{\left(1 - \frac{mc}{100}\right)} \times \frac{139}{\left(209 - \%O_2\right)}$$

where

$C_{S@7\%O_2}$	=	PM concentration at standard conditions, corrected to 7 percent oxygen
T_a	=	stack gas temperature at actual conditions, °K
T_{std}	=	stack gas temperature at standard conditions = 293°K
P_{std}	=	stack gas pressure at standard conditions = 760 mm of Hg
P_a	=	stack gas pressure at actual conditions, mm of Hg
mc	=	stack gas moisture content, %
$\%O_2$	=	stack gas oxygen content, %.

$$340 = C_a \times \frac{(121 + 273)}{293} \times \left(\frac{760}{760}\right) \times \left(\frac{1}{1 - \frac{12}{100}}\right) \times \frac{139}{(209 - 10)}$$

$$340 = 195\,C_a$$

$$C_a = 1745$$

The emission limit in units of concentration at the operating conditions of the PM CEMS is 17.45 mg/acm. This is the value that must be used for determining the confidence and tolerance interval half range percentages.

What are the results of the correlation analysis?

Table Y-16 summarizes the results of the correlation analysis. Only the logarithmic model satisfies all of the acceptance criteria for PS-11. The correlation coefficient for the exponential model (0.812) fails to meet the minimum allowable value of 0.85. The confidence and tolerance interval half range percentages for the linear model (11.3 and 35.6 percent, respectively) exceed the limits of 10 and 25 percent, respectively. For the power model, the confidence interval (14.3 percent) and tolerance interval (46.4 percent) half range percentages also exceed the maximum allowable values.

Table Y-16. Summary of Results for Facility C Correlation Test

Model type	Correlation equation	Correlation coefficient (r)	Confidence interval half range	Tolerance interval half range
Acceptance Criteria for PS-11		$ 0.85	# 10%	# 25%
Linear	í = 2.27 + 1.15*x*	0.898	11.3%	35.6%
Polynomial	í = -18.4 + 4.30*x* - 0.101x^2	0.987	5.56%	13.4%
Logarithmic	í = -24.4 + 16.8Ln(*x*)	0.959	7.32%	23.1%
Exponential	í = 5.26$e^{0.0785x}$	0.812	19.9%	66.5%
Power	í = 0.747$x^{1.20}$	0.908	14.3%	46.4%

The polynomial model has the highest correlation coefficient (0.987) and meets the criteria for confidence and tolerance interval half range percentages. However, Section 12.4(3) of PS-11 specifies that the maximum value for a polynomial correlation curve must occur at a PM CEMS response value that is at least 1.25 times the highest PM CEMS response value used to develop the correlation equation. In this example, the maximum PM CEMS response value is 24.7 mA, and the maximum value for the correlation curve must occur at a PM CEMS response value of at least 32.4 mA (24.7 × 1.25 = 30.88). However, the maximum value for the polynomial correlation curve occurs at 21.2 mA, as is evident from Figure Y-4. Therefore, the polynomial model cannot be used. The complete output for the spreadsheet for this example is provided in Appendix A.3.

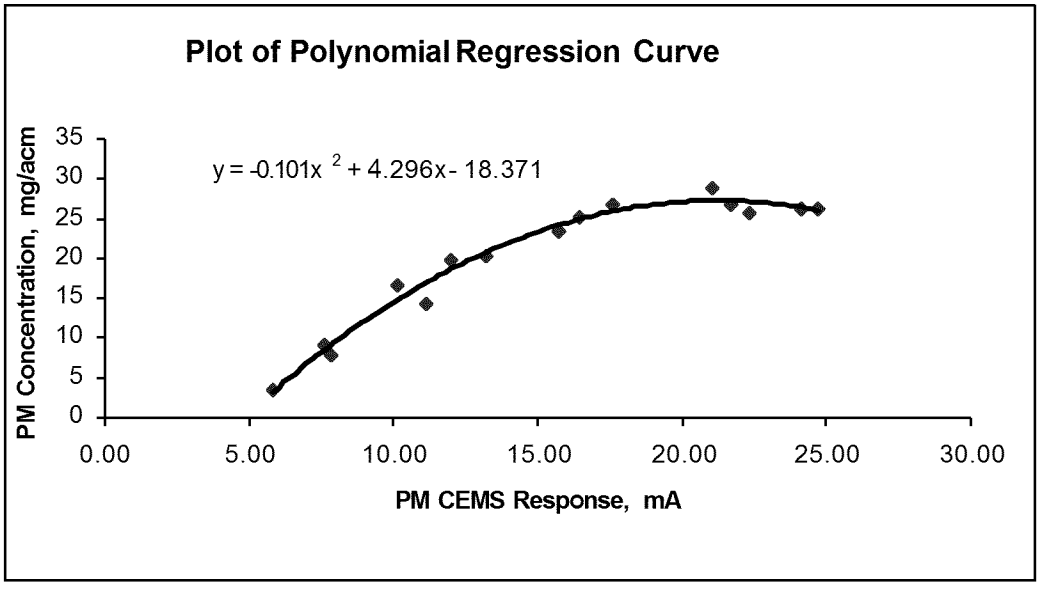


Figure Y-4. Plot of polynomial correlation curve for Facility C.

Y.4 EXAMPLE 4: WHAT CAN I DO IF MY PM CEMS FAILS THE CORRELATION TEST?

Y.4.1 Correlation Test Scenario

A correlation test is performed on a boiler at Facility D. The boiler is equipped with an in situ PM CEMS that responds to PM concentrations at actual conditions. The emission limit, when converted to units that are consistent with the PM CEMS sampling conditions, is 18.5 mg/acm. The correlation test consists of 15 test runs using a single Method 5 sampling train. Table Y-17 summarizes the correlation test data. Figure Y-5 shows a plot of the data.

Table Y-17. Summary of Test Data for Facility D

Run no.	PM CEMS response, mA	PM concentration, mg/acm
1	6.7	3.1
2	7.6	7.4
3	7.8	2.1
4	9.2	3.7
5	10.1	6.1
6	11.1	2.8
7	12.0	4.9
8	18.2	8.3
9	13.7	4.5
10	14.6	8.2
11	19.7	13.0
12	18.0	11.1
13	20.3	11.9
14	20.2	10.6
15	19.7	9.2

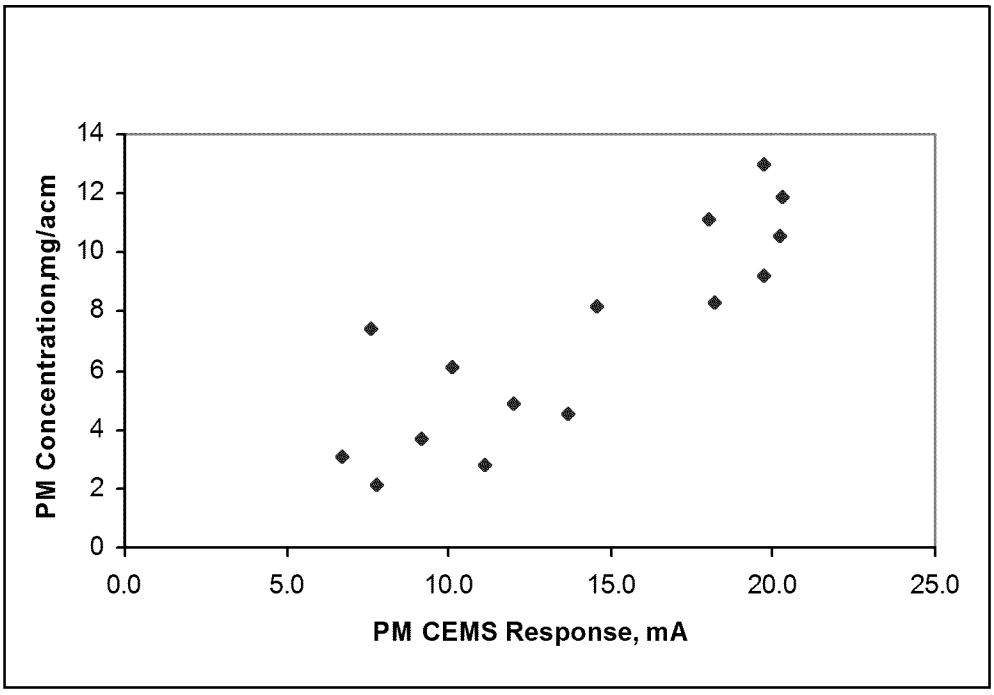


Figure Y-5. Plot of correlation test data for Facility D.

The results of the correlation analysis are summarized in Table Y-18. None of the models satisfy all of the correlation criteria specified in PS-11. Although all of the models meet the criteria for confidence and tolerance interval half range percentages, only the polynomial model has an acceptable correlation coefficient. However, the polynomial correlation curve has a minimum x value of 7.05 that is greater than the minimum PM CEMS response (6.7 mA for Run 1) value used to develop the correlation curve. As stated in Section 12.4(3)(iii) of PS-11, a polynomial correlation curve cannot be used if it has a minimum value that exceeds the minimum PM-CEMS response value used to determine the correlation curve. Consequently, the polynomial correlation also is unacceptable.

Table Y-18. Results of Initial Correlation for Facility D

Model type	Correlation equation	Correlation coefficient (r)	Confidence interval half range	Tolerance interval half range
Acceptance Criteria for PS-11		$ 0.85	# 10%	# 25%
Linear	í = -1.20 + 0.598x	0.841	5.80%	18.3%
Polynomial	í = 6.19 - 0.615 + 0.0436x^2	0.855	7.68%	18.2%
Logarithmic	í = -11.8 + 7.37Ln(x)	0.801	6.41%	20.2%
Exponential	í = 1.69$e^{0.0934x}$	0.806	6.41%	21.3%
Power	í = 0.308$x^{1.17}$	0.782	6.76%	22.6%

Several options are available for meeting the requirements of PS-11, including the following:

- Repeat the correlation test: If you suspect problems with the manual sampling or the operation of the PM CEMS, you may want to discard all of the test data and conduct a new correlation test. In such a case, there must be a valid reason for rejecting the entire data set. For example, you determine that several of the PM samples were contaminated. You still must report all of the rejected data.

- Conduct additional test runs. You may want to conduct a limited number of additional test runs (e.g., five additional runs) if it appears that some of the data are outliers, problems are suspected for some of the runs, or the range of PM concentrations measured was not adequate to fully characterize the correlation. In this case, you can develop the correlation based on all of the data (i.e., the initial 15 runs plus the additional runs), or you can reject some of the test runs, provided you meet the requirements of Section 8.6(3) of PS-11, which allows you to reject up to five test runs without explanation and additional runs if the basis for rejecting them is specified in the test method, in PS-11, or in your quality assurance plan.

- Relocate the PM CEMS and conduct a new correlation test: If the sampling location is not ideal (e.g., PM concentrations are stratified across the stack at the sampling point), you may want to consider relocating the PM CEMS to another location that is more suitable. In this case, you must conduct a full correlation test (i.e., at least 15 valid test runs).

- Replace the PM CEMS: Certain types of PM CEMS may not be suitable for specific stack gas conditions (e.g., an in situ light scattering instrument in a wet stack). If that is the case, you may decide to install a PM CEMS that uses a different measurement principle. If so, you must conduct a full correlation test with the new instrument.

- Consider alternative solutions: Section 12.4(4) allows you to petition the regulatory authority for approval of alternatives to the correlation requirements of PS-11.

For this example, we will assume that you decide to conduct five additional test runs. These runs are identified as Runs 16 to 20. Table Y-19 shows the results of the additional runs. Figure Y-6 shows the results of all 20 test runs.

Table Y-19. Results of Additional Test Runs

Run no.	PM CEMS response, mA	PM concentration, mg/acm
16	5.2	2.4
17	8.6	4.8
18	14.1	6.2
19	14.5	7.0
20	17.3	8.6

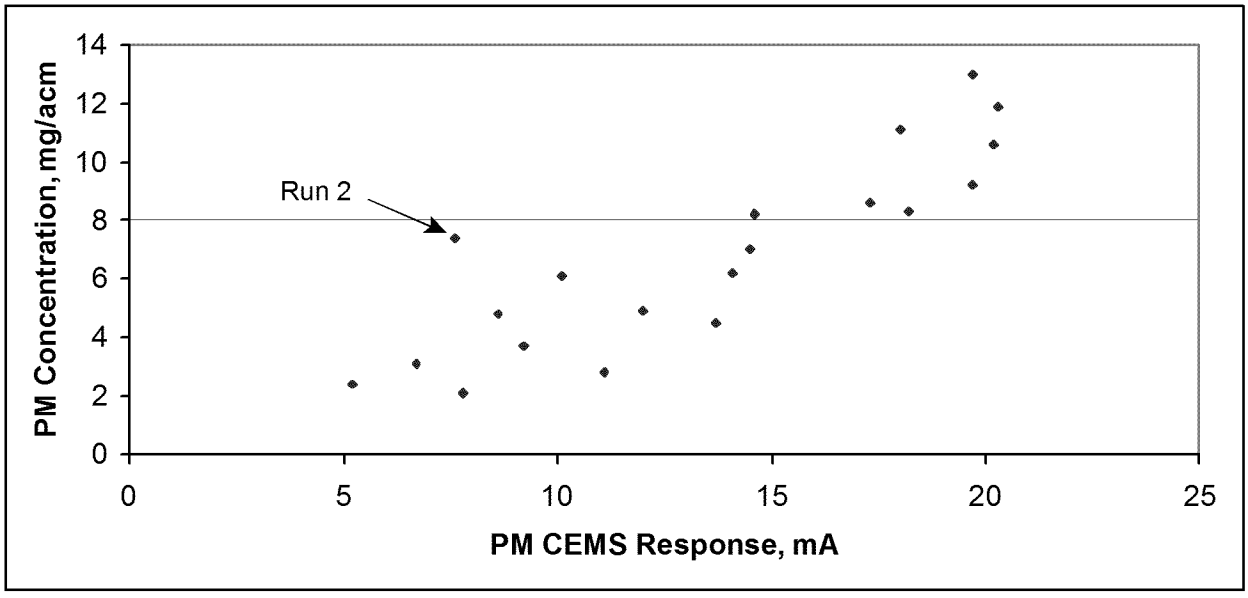


Figure Y-6. Plot of original 15 runs plus 5 additional runs for Facility D.

Table Y-20 shows the results of the correlation analysis on all 20 test runs. The analysis shows that the linear and polynomial models meet the correlation criteria of PS-11, and no further analyses or testing is required. With the additional data included, the minimum value for the polynomial correlation curve now occurs at 4.2 mA, which is less than the minimum PM CEMS value used to develop the curve (5.2 mA). However, a review of Figure Y-6 indicates that Run 2 (7.4 mA, 7.4 mg/acm) may be an outlier. You may decide to repeat the correlation analysis with Run 2 excluded. Table Y-21 shows the results of the analysis without the data from Run 2. In the revised analysis, all five models meet the criteria of PS-11. The correlation coefficients for the linear and polynomial models also are considerably greater for the revised analysis. You have the option of using the results of either analyses. In either case, the polynomial model provides the best fit; however, any of the models that satisfy all of the correlation criteria in PS-11 can be used.

Table Y-20. Results of Revised Correlation Analysis with All 20 Runs

Model type	Correlation equation	Correlation coefficient (r)	Confidence interval half range	Tolerance interval half range
Acceptance Criteria for PS-11		$ 0.85	# 10%	# 25%
Linear	í = -0.905 + 0.573x	0.861	4.25%	14.7%
Polynomial	í = 3.94 - 0.263x + 0.0314x^2	0.875	5.47%	14.4%
Logarithmic	í = -9.79 + 6.57Ln(x)	0.816	4.82%	16.7%
Exponential	í = 1.70$e^{0.0935x}$	0.840	4.52%	16.3%
Power	í = 0.367$x^{1.11}$	0.824	4.73%	17.0%

Table Y-21. Results of Revised Correlation Analysis with Run 2 Excluded

Model type	Correlation equation	Correlation coefficient (r)	Confidence interval half range	Tolerance interval half range
Acceptance Criteria for PS-11		$ 0.85	# 10%	# 25%
Linear	í = -1.88 + 0.629x	0.912	3.63%	12.4%
Polynomial	í = 2.19 - 0.067x + 0.026x^2	0.921	4.54%	12.1%
Logarithmic	í = -11.7 + 7.24Ln(x)	0.867	4.40%	15.0%
Exponential	í = 1.41e$^{0.104x}$	0.909	3.63%	12.6%
Power	í = 0.251$x^{1.24}$	0.894	3.90%	13.6%

The complete output of the spreadsheet for the initial correlation, the correlation with all five additional test runs, and the correlation with Run 2 excluded are provided in Appendices A.4 through A.6.

Appendix A.1

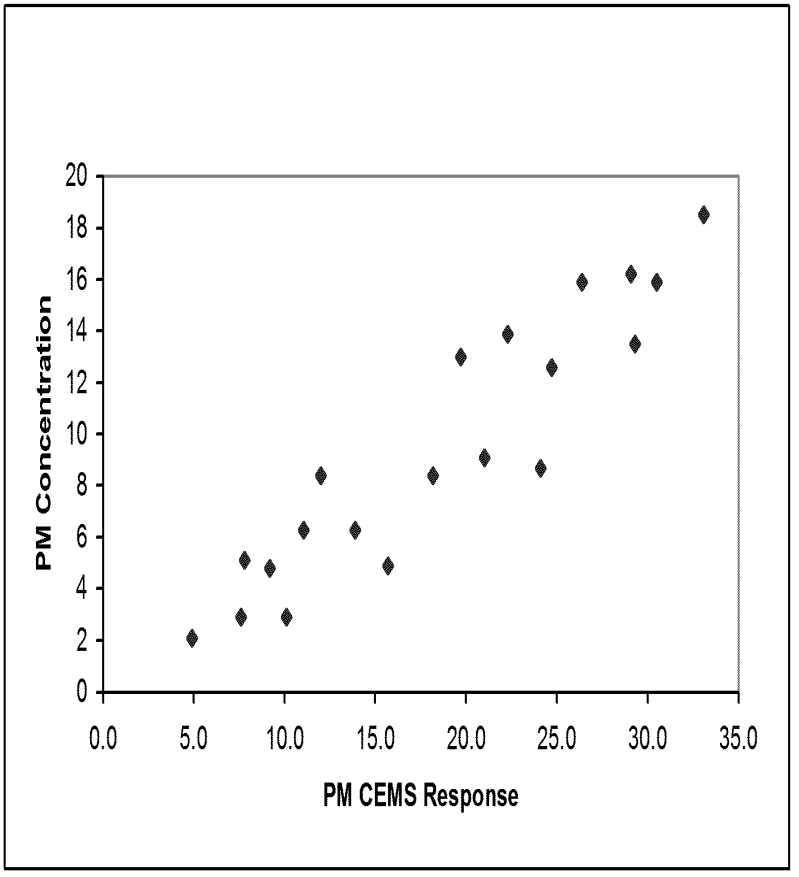
Complete Results of Example No. 1 – Initial Correlation Test

CORRELATION TEST PM CEMS AND REFERENCE METHOD TEST DATA

Run	PM CEMS response x	PM concentration mg/acm y
1	4.9	2.1
2	7.6	2.9
3	7.8	5.1
4	9.2	4.8
5	10.1	2.9
6	11.1	6.3
10	12.0	8.4
11	18.2	8.4
12	15.7	4.9
13	13.9	6.3
14	19.7	13.0
15	21.0	9.1
16	22.3	13.9
17	24.1	8.7
19	24.7	12.6
20	26.4	15.9
21	29.1	16.2
22	29.3	13.5
23	30.5	15.9
24	33.1	18.5

Facility: Facility A
Location: Anytown USA
Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

	Value	Units
Emission limit:	20	mg/acm



CALCULATIONS FOR LINEAR CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

Run	PM CEMS response x	PM concentration mg/acm y	CALCULATED STATISTICAL PARAMETERS				
			$(x_i-\tilde{x})^2$	$(y_i-\tilde{y})^2$	$(x_i-\tilde{x})(y_i-\tilde{y})$	$\hat{y}$	$(\hat{y}_i-y_i)^2$
1	4.9	2.1	185.86	54.32	100.48	2.047	0.003
2	7.6	2.9	119.53	43.16	71.83	3.517	0.381
3	7.8	5.1	115.20	19.10	46.90	3.626	2.172
4	9.2	4.8	87.10	21.81	43.59	4.388	0.169
5	10.1	2.9	71.12	43.16	55.40	4.878	3.914
6	11.06	6.3	55.85	10.05	23.69	5.401	0.808
10	12	8.4	42.68	1.14	6.99	5.913	6.185
11	18.2	8.4	0.11	1.14	0.36	9.289	0.790
12	15.7	4.9	8.03	20.88	12.95	7.928	9.166
13	13.9	6.3	21.46	10.05	14.69	6.947	0.419
14	19.7	13	1.36	12.46	4.12	10.105	8.379
15	21.02	9.1	6.19	0.14	-0.92	10.824	2.973
16	22.3	13.9	14.19	19.62	16.69	11.521	5.660
17	24.1	8.7	30.99	0.59	-4.29	12.501	14.448
19	24.7	12.6	38.03	9.80	19.30	12.828	0.052
20	26.4	15.9	61.89	41.34	50.58	13.753	4.608
21	29.08	16.2	111.24	45.29	70.98	15.213	0.975
22	29.3	13.5	115.93	16.24	43.39	15.332	3.357
23	30.5	15.9	143.21	41.34	76.95	15.986	0.007
24	33.1	18.5	212.20	81.54	131.54	17.401	1.207

RESULTS OF LINEAR CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

Variable	Equation	Value
n	Number of data points =	20
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	18.53
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	1,442
$\tilde{y}$	y~ = 1/n*(Sum of (y_i)) =	9.47
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	493
S_{xy}	S_{xy} = Sum((x_i-x~)(y_i-y~)) =	785.2
b_0	b_0 = y~-b_1x~ =	-0.621
b_1	b_1 = S_{xy}/S_{xx} =	0.544
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	1.910
$y^{\wedge}_{mean}$	y^ at mean x value =	9.470
t_f	t_f from table =	2.101
CI	CI = t_f*S_L*SQRT(1/n) =	0.897
EL	Emission Limit =	20.0
CI%	CI% =CI/EL*100% =	4.49%
n'	n' = n =	20
v_f	v_f from table =	1.384
$u_{n'}$	$u_{n'}$ from table =	1.177
k_T	k_t = u_n'*v_f =	1.629
TI	TI = k_t*S_L =	3.111
TI%	TI% = TI/EL*100% =	15.6%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	5.095
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.859
r	r = SQRT((1-S_L^2/S_y^2)) =	0.927

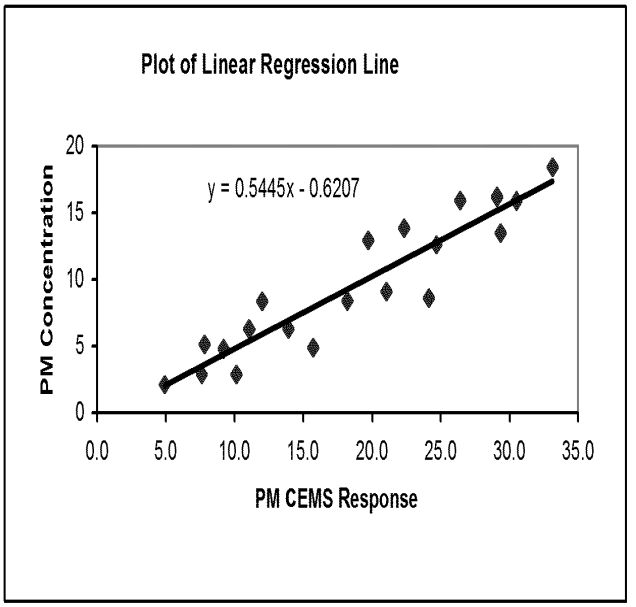
* Indicates correlation coefficient is undefined.

Correlation equation: y = -0.621 + 0.544 x

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.927	≥ 0.85	yes
Confidence interval	4.49%	≤ 10%	yes
Tolerance interval	15.6%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR POLYNOMIAL CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

	PM CEMS response	PM concentration mg/acm	CALCULATED STATISTICAL PARAMETERS									
Run	x	y	x^2	x^3	x^4	xy	x^2y	y^	$(y^\wedge-y)^2$	delta	$(yi-y\sim)^2$	CI
1	4.9	2.1	24	118	576	10	50	2.497	0.157	0.3817	54.32	2.54
2	7.6	2.9	58	439	3,336	22	168	3.717	0.667	0.1727	43.16	1.71
3	7.8	5.1	61	475	3,702	40	310	3.809	1.667	0.1634	19.10	1.66
4	9.2	4.8	85	779	7,164	44	406	4.465	0.112	0.1162	21.81	1.40
5	10.1	2.9	102	1,030	10,406	29	296	4.894	3.975	0.0995	43.16	1.30
6	11.06	6.3	122	1,353	14,963	70	771	5.358	0.887	0.0906	10.05	1.24
10	12	8.4	144	1,728	20,736	101	1,210	5.819	6.660	0.0884	1.14	1.22
11	18.2	8.4	331	6,029	109,720	153	2,782	9.024	0.390	0.1202	1.14	1.43
12	15.7	4.9	246	3,870	60,757	77	1,208	7.698	7.828	0.1084	20.88	1.36
13	13.9	6.3	193	2,686	37,330	88	1,217	6.771	0.222	0.0960	10.05	1.27
14	19.7	13	388	7,645	150,614	256	5,045	9.842	9.972	0.1204	12.46	1.43
15	21.02	9.1	442	9,287	195,223	191	4,021	10.576	2.178	0.1162	0.14	1.40
16	22.3	13.9	497	11,090	247,297	310	6,912	11.299	6.764	0.1092	19.62	1.36
17	24.1	8.7	581	13,998	337,340	210	5,053	12.337	13.229	0.0985	0.59	1.29
19	24.7	12.6	610	15,069	372,210	311	7,687	12.688	0.008	0.0959	9.80	1.27
20	26.4	15.9	697	18,400	485,753	420	11,082	13.698	4.849	0.0960	41.34	1.27
21	29.08	16.2	846	24,591	715,118	471	13,699	15.333	0.752	0.1417	45.29	1.55
22	29.3	13.5	858	25,154	737,005	396	11,590	15.469	3.879	0.1492	16.24	1.59
23	30.5	15.9	930	28,373	865,365	485	14,791	16.221	0.103	0.2046	41.34	1.86
24	33.1	18.5	1,096	36,265	1,200,361	612	20,269	17.884	0.379	0.4312	81.54	2.70

RESULTS OF POLYNOMIAL CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

Variable	Equation	Value
n	Number of data points =	20
S_1	S1 = Sum (xi) =	371
S_2	S2 = Sum (xi2) =	8,312
S_3	S3 = Sum (xi3) =	208,377
S_4	S4 = Sum (xi4) =	5,574,977
S_5	S5 = Sum (yi) =	189
S_6	S6 = Sum (xiyi) =	4,295
S_7	S7 = Sum (xi2yi) =	108,567
det A	det A = nS2S4-S2S2S2+S1S3S2-S3S3n+S2S1S3-S4S1S1 =	2.120E+09
b_0	b0 = (S5S2S4+S1S3S7+S2S6S3-S7S2S2-S3S3S5-S4S6S1)/det A =	0.420
b_1	b1 = (nS6S4+S5S3S2+S2S1S7-S2S6S2-S7S3n-S4S1S5)/det A =	0.406
b_2	b2 = (nS2S7+S1S6S2+S5S1S3-S2S2S5-S3S6n-S7S1S1)/det A =	0.00368
S_p	S_p = Sqrt((1/(n-3)Sum of (y^-Y)2) =	1.951
D	D = n(S2S4-S3^2)+S1(S3S2-S1S4)+S2(S1S3-S2^2) =	2.120E+09
C_0	C0 = (S2S4-S3^2)/D =	1.375
C_1	C1 = (S3S2-S1S4)/D =	-0.1578
C_2	C2 = (S1S3-S2^2)/D =	3.846E-03
C_3	C3 = (nS4-S2^2)/D =	2.001E-02
C_4	C4 = (S1S2-nS3)/D =	-5.126E-04
C_5	C5 = (nS2-S1^2)/D =	1.360E-05
t_f	t_f from table =	2.110
EL	Emission Limit =	20.0
CI	CI = t_f*S_p *sqrt(delta-min) =	1.223
CI%	CI% = CI/EL*100=	6.12%
v_f	v_f from table =	1.400
$u_{n'}$	$u_{n'}$ from table =	1.195
n'	n' = 1/delta =	11.31
k_T	kt = un'*vf =	1.673
TI	TI = kt*sp =	3.263
TI%	TI% = TI/EL *100% =	16.3%
y~	y~ = 1/n*(Sum of (yi)) =	9.47
S_y	Sy = sqrt(Sum of (yi-y~)^2/(n-1) =	5.09
r^2	r^2 = 1-(S_p^2/Sy^2) =	0.853
r	r = sqrt((1-S_p^2/Sy^2)) =	0.924
Max-min?	b_2 > 0?	minimum
$x_{max-min}$	y = -b_1/2b_2 =	-55.08
1.25x_{max}		41.38

* Indicates correlation coefficient is undefined.

Correlation equation: $y = 0.420 + 0.406\ x + 0.00368\ x^2$

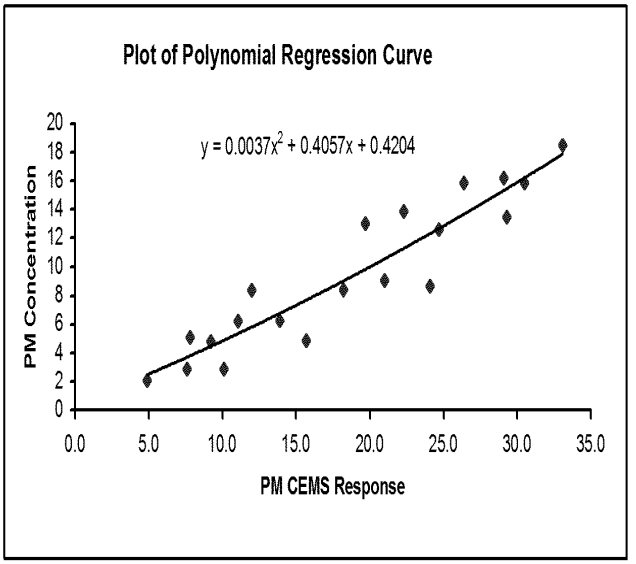
Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.924	≥ 0.85	yes
Confidence interval	6.12%	≤ 10%	yes
Tolerance interval	16.3%	≤ 25%	yes

* Indicates correlation coefficient is undefined.

Check for Correlation Curve Minimum/Maximum

Correlation curve minimum point	-55.08
Minimum allowable x value	4.9
Is correlation curve minimum < minimum x value?	yes
Correlation curve maximum point	NA
Extrapolation limit for x (125% of maximum x value)	NA
Is correlation curve maximum > extrapolation limit?	NA



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CALCULATIONS FOR LOGARITHMIC CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

	PM CEMS response		PM concentration	CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	mg/acm					
Run	x	x' = ln (x)	y	$(x_i-\bar{x'})^2$	$(y_i-\bar{y})^2$	$(x'_i-\bar{x'})(y_i-\bar{y})$	y^	$(y\hat{}_i-y_i)^2$
1	4.9	1.589	2.1	1.446	54.317	8.863	-0.539	6.964
2	7.6	2.028	2.9	0.583	43.165	5.017	3.114	0.046
3	7.8	2.054	5.1	0.544	19.097	3.224	3.330	3.132
4	9.2	2.219	4.8	0.328	21.809	2.674	4.704	0.009
5	10.1	2.313	2.9	0.230	43.165	3.149	5.481	6.661
6	11.06	2.403	6.3	0.151	10.049	1.231	6.237	0.004
10	12	2.485	8.4	0.094	1.145	0.328	6.916	2.203
11	18.2	2.901	8.4	0.012	1.145	-0.117	10.382	3.929
12	15.7	2.754	4.9	0.001	20.885	0.174	9.152	18.083
13	13.9	2.632	6.3	0.026	10.049	0.507	8.139	3.382
14	19.7	2.981	13	0.036	12.461	0.666	11.041	3.836
15	21.02	3.045	9.1	0.064	0.137	-0.094	11.581	6.156
16	22.3	3.105	13.9	0.098	19.625	1.386	12.073	3.338
17	24.1	3.182	8.7	0.152	0.593	-0.301	12.719	16.154
19	24.7	3.207	12.6	0.172	9.797	1.299	12.924	0.105
20	26.4	3.273	15.9	0.232	41.345	3.096	13.478	5.867
21	29.08	3.370	16.2	0.334	45.293	3.891	14.283	3.677
22	29.3	3.378	13.5	0.343	16.241	2.361	14.345	0.714
23	30.5	3.418	15.9	0.392	41.345	4.025	14.679	1.490
24	33.1	3.500	18.5	0.501	81.541	6.391	15.360	9.858

RESULTS OF LOGARITHMIC CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

Variable	Equation	Value
n	Number of data points =	20
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.79
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	5.74
$y^{\sim}$	y~ = 1/n*(Sum of (y_i)) =	9.5
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	493
$S_{x'y}$	$S_{x'y}$ = Sum((x'_i-x'~)(y_i-y~)) =	47.8
b_0	b_0 = y~-b_1x'~ =	-13.766
b_1	b_1 = $S_{x'y}$/$S_{x'x'}$ =	8.323
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	2.305
$y^{\wedge}_{mean}$	y^ at mean x value =	9.470
t_f	t_f from table =	2.101
CI	CI = t_f*S_L*SQRT(1/n) =	1.083
EL	Emission Limit =	20.0
CI%	CI% =CI/EL*100% =	5.41%
n'	n' = n =	20
v_f	v_f from table =	1.384
$u_{n'}$	$u_{n'}$ from table =	1.177
k_T	k_t = u_n'*v_f =	1.629
TI	TI = k_t*SL =	3.75
TI%	TI% = TI/EL*100% =	18.8%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	5.095
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.795
r	r = SQRT((1-S_L^2/S_y^2)) =	0.892

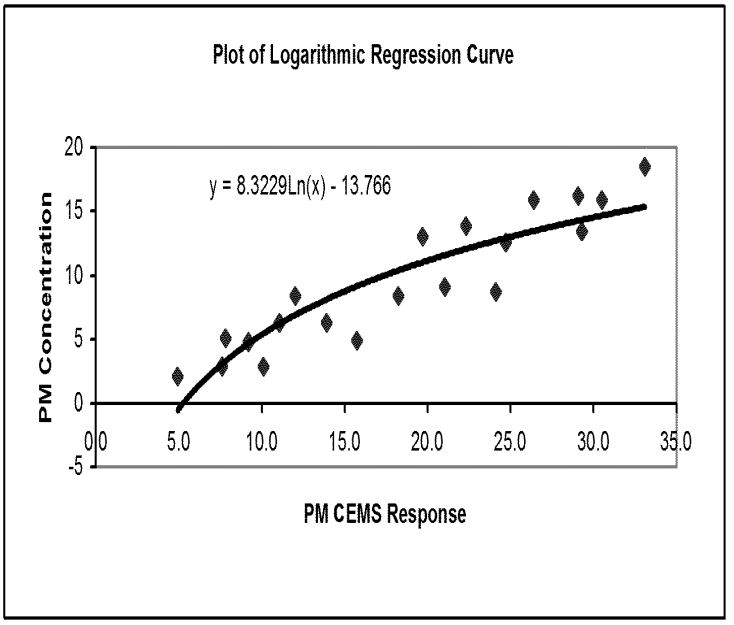
* Indicates correlation coefficient is undefined.

Correlation equation: y = -13.766 + 8.323 Ln(x)

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.892	≥ 0.85	yes
Confidence interval	5.41%	≤ 10%	yes
Tolerance interval	18.8%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
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CALCULATIONS FOR EXPONENTIAL CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

Run	PM CEMS response	PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
		Measured	Transformed					
	x	y	y' = Ln(y)	$(x_i-\bar{x})^2$	$(y'_i-\bar{y'})^2$	$(x_i-\bar{x})(y'_i-\bar{y'})$	y'^	$(y'^{\wedge}_i-y'_i)^2$
1	4.9	2.1	0.742	185.86	1.782	18.198	1.164	0.178
2	7.6	2.9	1.065	119.53	1.024	11.065	1.345	0.078
3	7.8	5.1	1.629	115.20	0.200	4.804	1.358	0.074
4	9.2	4.8	1.569	87.10	0.258	4.743	1.452	0.014
5	10.1	2.9	1.065	71.12	1.024	8.535	1.512	0.200
6	11.06	6.3	1.841	55.85	0.056	1.766	1.576	0.070
10	12	8.4	2.128	42.68	0.003	-0.336	1.639	0.239
11	18.2	8.4	2.128	0.11	0.003	-0.017	2.055	0.005
12	15.7	4.9	1.589	8.03	0.238	1.381	1.887	0.089
13	13.9	6.3	1.841	21.46	0.056	1.095	1.767	0.005
14	19.7	13	2.565	1.36	0.238	0.570	2.155	0.168
15	21.02	9.1	2.208	6.19	0.017	0.327	2.243	0.001
16	22.3	13.9	2.632	14.19	0.308	2.091	2.329	0.092
17	24.1	8.7	2.163	30.99	0.007	0.482	2.450	0.082
19	24.7	12.6	2.534	38.03	0.209	2.818	2.490	0.002
20	26.4	15.9	2.766	61.89	0.475	5.424	2.604	0.026
21	29.08	16.2	2.785	111.24	0.502	7.469	2.783	0.000
22	29.3	13.5	2.603	115.93	0.277	5.662	2.798	0.038
23	30.5	15.9	2.766	143.21	0.475	8.251	2.878	0.013
24	33.1	18.5	2.918	212.20	0.707	12.250	3.052	0.018

RESULTS OF EXPONENTIAL CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

Variable	Equation	Value
n	Number of data points =	20
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	18.53
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	1,442.2
$y'^{\sim}$	y'~ = 1/n*(Sum of (y'_i)) =	2.077
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	7.86
$S_{xy'}$	$S_{xy'}$ = Sum((x_i-x~)(y'_i-y'~)) =	96.58
b_0'	b_0' = y'~-b_1x~ =	0.836
b_0	b_0 = e^b_0' =	2.306
b_1	b_1 = $S_{xy'}$/S_{xx} =	0.067
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.278
$y'^{\wedge}_{mean}$	y'^ at mean x value =	2.077
t_f	t_f from table =	2.101
CI'	CI' = t_f*SL*SQRT(1/n) =	0.1307
LCL'	LCL' = y~' - CI'	1.946
UCL'	UCL' = y~' + CI'	2.207
CI	CI = (e^UCL' - e^LCL')/2	1.045
EL	Emission Limit =	20.0
CI%	CI% =CI/EL*100% =	5.23%
n'	n' = n =	20
v_f	v_f from table =	1.384
$u_{n'}$	$u_{n'}$ from table =	1.177
k_T	k_T = u_n'*v_f =	1.629
TI'	TI' = k_t*SL =	0.453
LTL'	LTL' = y~' - TI'	1.624
UTL'	UTL' = y~' + TI'	2.530
TI	TI = (e^UTL' - e^LTL')/2	3.740
TI%	TI% = TI/EL*100% =	18.7%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.643
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.813
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.902

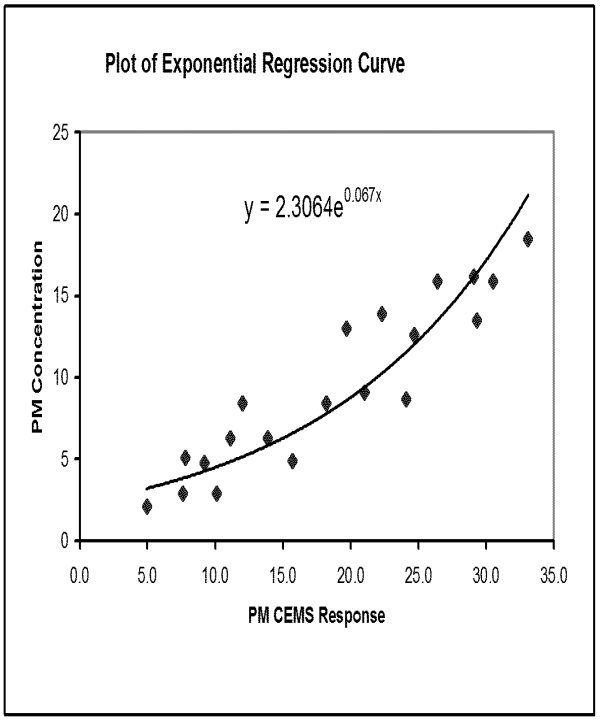
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 2.306\ e^{0.0670\ x}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.902	≥ 0.85	yes
Confidence interval	5.23%	≤ 10%	yes
Tolerance interval	18.7%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
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CALCULATIONS FOR POWER CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

	PM CEMS response		PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	Measured	Transformed					
Run	x	x' = Ln(x)	y	y' = Ln(y)	$(x'_i-x'^{\sim})^2$	$(y'_i-y'^{\sim})^2$	$(x'_i-x'^{\sim})(y'_i-y'^{\sim})$	y'^	$(y'_i{}^{\wedge}-y'_i)^2$
1	4.9	1.589	2.1	0.742	1.446	1.782	1.605	0.773	0.0010
2	7.6	2.028	2.9	1.065	0.583	1.024	0.773	1.249	0.0340
3	7.8	2.054	5.1	1.629	0.544	0.200	0.330	1.277	0.1239
4	9.2	2.219	4.8	1.569	0.328	0.258	0.291	1.456	0.0126
5	10.1	2.313	2.9	1.065	0.230	1.024	0.485	1.557	0.2427
6	11.06	2.403	6.3	1.841	0.151	0.056	0.092	1.656	0.0342
10	12	2.485	8.4	2.128	0.094	0.003	-0.016	1.744	0.1475
11	18.2	2.901	8.4	2.128	0.012	0.003	0.006	2.196	0.0045
12	15.7	2.754	4.9	1.589	0.001	0.238	0.019	2.035	0.1991
13	13.9	2.632	6.3	1.841	0.026	0.056	0.038	1.903	0.0040
14	19.7	2.981	13	2.565	0.036	0.238	0.092	2.281	0.0804
15	21.02	3.045	9.1	2.208	0.064	0.017	0.033	2.352	0.0206
16	22.3	3.105	13.9	2.632	0.098	0.308	0.174	2.416	0.0467
17	24.1	3.182	8.7	2.163	0.152	0.007	0.034	2.500	0.1133
19	24.7	3.207	12.6	2.534	0.172	0.209	0.190	2.527	0.0001
20	26.4	3.273	15.9	2.766	0.232	0.475	0.332	2.599	0.0281
21	29.08	3.370	16.2	2.785	0.334	0.502	0.410	2.704	0.0066
22	29.3	3.378	13.5	2.603	0.343	0.277	0.308	2.712	0.0119
23	30.5	3.418	15.9	2.766	0.392	0.475	0.432	2.755	0.0001
24	33.1	3.500	18.5	2.918	0.501	0.707	0.595	2.844	0.0055

RESULTS OF POWER CORRELATION

Facility: Facility A
Location: Anytown USA

Emission Unit: Boiler No. 3
Test Dates: March 6 to 15, 2004

Variable	Equation	Value
n	Number of data points =	20
$x'^{\sim}$	$x'\sim = 1/n*(\text{Sum of } (x'_i)) =$	2.792
$S_{x'x'}$	$S_{x'x'} = \text{Sum}((x'_i-x'\sim)^2) =$	5.74
$y'^{\sim}$	$y'\sim = 1/n*(\text{Sum of } (y'_i)) =$	2.077
$S_{y'y'}$	$S_{y'y'} = \text{Sum}((y'_i-y'\sim)^2) =$	7.86
$S_{x'y'}$	$S_{x'y'} = \text{Sum}((x'_i-x'\sim)(y'_i-y'\sim)) =$	6.22
b_0'	$b_0' = y'\sim-b_1x\sim =$	-0.949
b_0	$b_0 = e^{b_0'} =$	0.387
b_1	$b_1 = S_{x'y'}/S_{x'x'} =$	1.084
S_L	$S_L = \text{SQRT}(1/(n-2)(\text{Sum}(y'^{\wedge}_i-y'_i)^2)) =$	0.249
$y'^{\wedge}_{mean}$	$y'^{\wedge}$ at mean x value =	2.077
t_f	t_f from table =	2.101
CI'	$CI = t_f*SL*\text{SQRT}(1/n) =$	0.117
LCL'	$LCL' = y\sim' - CI'$	1.960
UCL'	$UCL' = y\sim' + CI'$	2.194
CI	$CI = (e^{UCL'} - e^{LCL'})/2$	0.936
EL	Emission Limit =	20.0
CI%	$CI\% = CI/EL*100\% =$	4.68%
n'	$n' = n =$	20
v_f	v_f from table =	1.384
$u_{n'}$	$u_{n'}$ from table =	1.177
k_T	$k_T = u_{n'}*v_f =$	1.629
TI'	$TI' = k_TS_L =$	0.406
LTL'	$LTL' = y\sim' - TI'$	1.671
UTL'	$UTL' = y\sim' + TI'$	2.483
TI	$TI = (e^{UTL'} - e^{LTL'})/2$	3.327
TI%	$TI\% = TI/EL*100\% =$	16.6%
$S_{y'}$	$S_{y'} = \text{SQRT}(S_{y'y'}/(n-1)) =$	0.643
r^2	$r^2 = 1-(SL^2/S_{y'}^2) =$	0.850
r	$r = \text{SQRT}((1-SL^2/S_{y'}^2)) =$	0.922

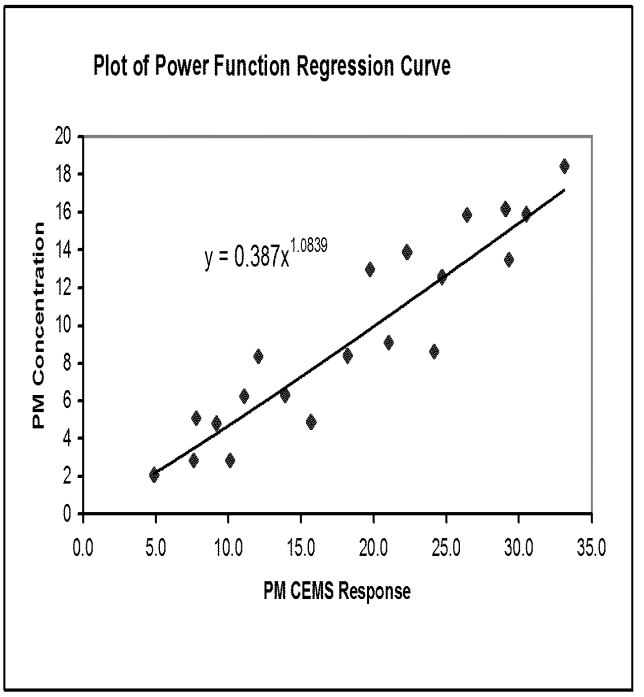
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 0.3870\ x^{1.0839}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.922	$\geq$ 0.85	yes
Confidence interval	4.68%	$\leq$ 10%	yes
Tolerance interval	16.6%	$\leq$ 25%	yes

* Indicates correlation coefficient is undefined.



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SELECTION OF BEST MODEL

Facility: **Facility A** **Emission Unit:** **Boiler No. 3**
Location: **Anytown USA** **Test Dates:** **March 6 to 15, 2004**

Model	Correlation coefficient (a)	≥ 0.85 ?	Confidence interval half range percentage	≤ 10% ?	Tolerance interval half range percentage	≤ 25% ?	Min/max within allowable range?	Does model meet all criteria?
Linear	0.927	Yes	4.49%	Yes	15.6%	Yes	(b)	Yes
Polynomial	0.924	Yes	6.12%	Yes	16.3%	Yes	Yes	Yes
Logarithmic	0.892	Yes	5.41%	Yes	18.8%	Yes	(b)	Yes
Exponential	0.902	Yes	5.23%	Yes	18.7%	Yes	(b)	Yes
Power	0.922	Yes	4.68%	Yes	16.6%	Yes	(b)	Yes

(a) * indicates correlation coefficient is undefined; model does not satisfy criterion.
(b) Not applicable; criterion applies only to polynomial model.

Best model: **Linear**

PS-11 Correlation Test
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Appendix A.2

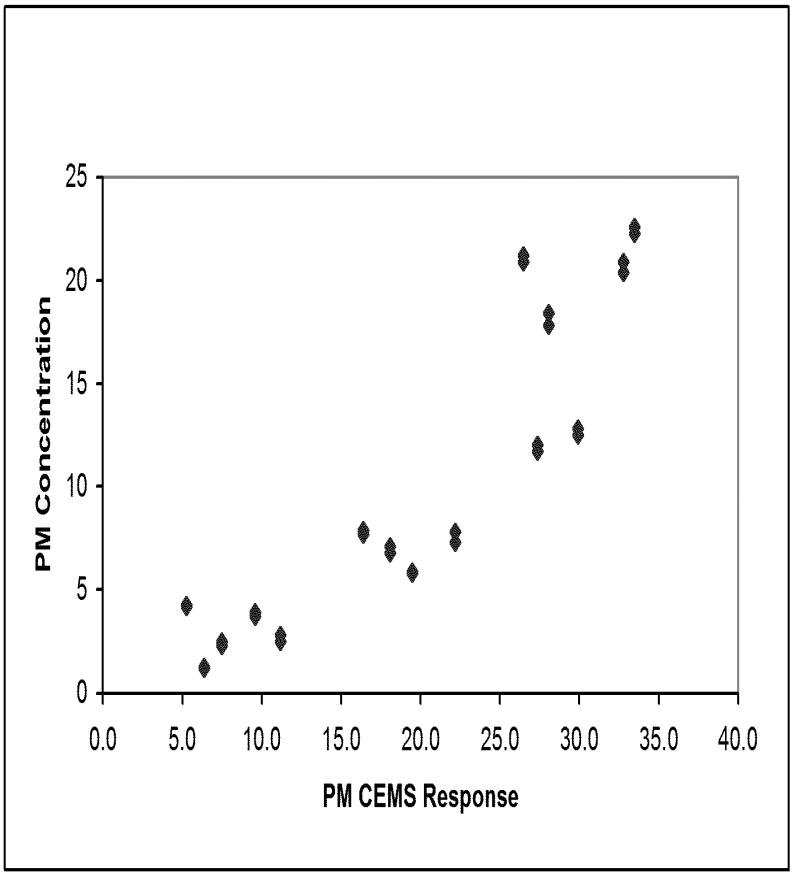
Complete Results of Example No. 2 – Initial Correlation Test

CORRELATION TEST PM CEMS AND REFERENCE METHOD TEST DATA

Run	PM CEMS response x	PM concentration mg/acm y
1A	6.4	1.3
1B	6.4	1.2
2A	7.5	2.3
2B	7.5	2.5
3A	11.2	2.5
3B	11.2	2.8
4A	16.4	7.7
4B	16.4	7.9
5A	29.9	12.9
5B	29.9	12.5
6A	22.2	7.8
6B	22.2	7.3
7A	19.5	5.8
7B	19.5	5.9
8A	27.4	11.7
8B	27.4	12
9A	32.8	20.4
9B	32.8	20.9
10A	26.5	20.9
10B	26.5	21.3
11A	33.5	22.3
11B	33.5	22.6
12A	28.1	18.4
12B	28.1	17.8
13A	9.6	3.7
13B	9.6	3.9
14A	5.3	4.2
14B	5.3	4.3
15A	18.1	6.8
15B	18.1	7.1

Facility: Facility B
Location: Anytown USA
Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

	Value	Units
Emission limit:	24.3	mg/acm



CALCULATIONS FOR LINEAR CORRELATION

Facility: Facility B
Location: Anytown USA
Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

Run	PM CEMS response x	PM concentration mg/acm y	CALCULATED STATISTICAL PARAMETERS				
			$(x_i-\tilde{x})^2$	$(y_i-\tilde{y})^2$	$(x_i-\tilde{x})(y_i-\tilde{y})$	$\hat{y}$	$(\hat{y}_i-y_i)^2$
1A	6.4	1.3	174.94	74.94	114.50	1.065	0.055
1B	6.4	1.2	174.94	76.68	115.82	1.065	0.018
2A	7.5	2.3	147.06	58.62	92.85	1.805	0.245
2B	7.5	2.5	147.06	55.60	90.42	1.805	0.483
3A	11.2	2.5	71.01	55.60	62.83	4.292	3.211
3B	11.2	2.8	71.01	51.22	60.31	4.292	2.226
4A	16.4	7.7	10.41	5.09	7.28	7.788	0.008
4B	16.4	7.9	10.41	4.23	6.64	7.788	0.013
5A	29.9	12.9	105.54	8.66	30.24	16.863	15.703
5B	29.9	12.5	105.54	6.47	26.13	16.863	19.034
6A	22.2	7.8	6.62	4.65	-5.55	11.687	15.105
6B	22.2	7.3	6.62	7.06	-6.84	11.687	19.242
7A	19.5	5.8	0.02	17.28	0.53	9.872	16.577
7B	19.5	5.9	0.02	16.46	0.51	9.872	15.773
8A	27.4	11.7	60.42	3.04	13.55	15.182	12.126
8B	27.4	12	60.42	4.18	15.88	15.182	10.126
9A	32.8	20.4	173.54	109.06	137.57	18.812	2.521
9B	32.8	20.9	173.54	119.76	144.16	18.812	4.359
10A	26.5	20.9	47.24	119.76	75.22	14.577	39.978
10B	26.5	21.3	47.24	128.67	77.97	14.577	45.197
11A	33.5	22.3	192.47	152.36	171.24	19.283	9.103
11B	33.5	22.6	192.47	159.85	175.41	19.283	11.004
12A	28.1	18.4	71.80	71.29	71.54	15.653	7.547
12B	28.1	17.8	71.80	61.52	66.46	15.653	4.611
13A	9.6	3.7	100.53	39.15	62.73	3.216	0.234
13B	9.6	3.9	100.53	36.68	60.73	3.216	0.467
14A	5.3	4.2	205.25	33.14	82.47	0.326	15.010
14B	5.3	4.3	205.25	32.00	81.04	0.326	15.794
15A	18.1	6.8	2.33	9.96	4.82	8.930	4.539
15B	18.1	7.1	2.33	8.16	4.36	8.930	3.350

RESULTS OF LINEAR CORRELATION

Facility: Facility B
Location: Anytown USA

Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

Variable	Equation	Value
n	Number of data points =	30
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	19.63
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	2,738
$\tilde{y}$	y~ = 1/n*(Sum of (y_i)) =	9.96
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	1,531
S_{xy}	S_{xy} = Sum((x_i-x~)(y_i-y~)) =	1,840.8
b_0	b_0 = y~-b_1x~ =	-3.237
b_1	b_1 = S_{xy}/S_{xx} =	0.672
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	3.238
$y^{\wedge}_{mean}$	y^ at mean x value =	9.957
t_f	t_f from table =	2.048
CI	CI = t_f*S_L*SQRT(1/n) =	1.211
EL	Emission Limit =	24.3
CI%	CI% =CI/EL*100% =	4.98%
n'	n' = n =	30
v_f	v_f from table =	1.286
$u_{n'}$	$u_{n'}$ from table =	1.168
k_T	k_t = u_n'*v_f =	1.503
TI	TI = k_t*S_L =	4.866
TI%	TI% = TI/EL*100% =	20.0%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	7.266
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.801
r	r = SQRT((1-S_L^2/S_y^2)) =	0.895

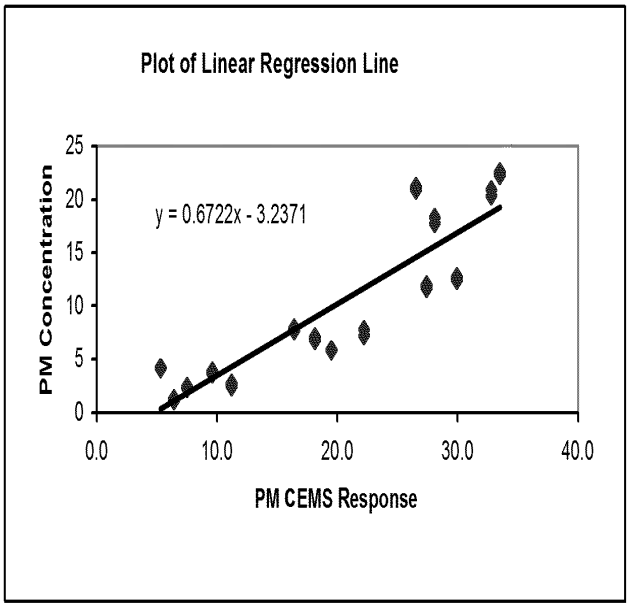
* Indicates correlation coefficient is undefined.

Correlation equation: y = -3.237 + 0.672 x

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.895	≥ 0.85	yes
Confidence interval	4.98%	≤ 10%	yes
Tolerance interval	20.0%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR POLYNOMIAL CORRELATION

Facility: Facility B
Location: Anytown USA

Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

	PM CEMS response	PM concentration mg/acm	CALCULATED STATISTICAL PARAMETERS									
Run	x	y	x^2	x^3	x^4	xy	x^2y	y^	$(y^\wedge-y)^2$	delta	$(yi-y\sim)^2$	CI
1A	6.4	1.3	41	262	1,678	8	53	2.649	1.819	0.1326	74.94	2.15
1B	6.4	1.2	41	262	1,678	8	49	2.649	2.099	0.1326	76.68	2.15
2A	7.5	2.3	56	422	3,164	17	129	2.795	0.245	0.1009	58.62	1.87
2B	7.5	2.5	56	422	3,164	19	141	2.795	0.087	0.1009	55.60	1.87
3A	11.2	2.5	125	1,405	15,735	28	314	3.680	1.392	0.0646	55.60	1.50
3B	11.2	2.8	125	1,405	15,735	31	351	3.680	0.774	0.0646	51.22	1.50
4A	16.4	7.7	269	4,411	72,339	126	2,071	5.942	3.090	0.0852	5.09	1.72
4B	16.4	7.9	269	4,411	72,339	130	2,125	5.942	3.833	0.0852	4.23	1.72
5A	29.9	12.9	894	26,731	799,254	386	11,533	17.379	20.061	0.0756	8.66	1.62
5B	29.9	12.5	894	26,731	799,254	374	11,175	17.379	23.804	0.0756	6.47	1.62
6A	22.2	7.8	493	10,941	242,891	173	3,844	9.872	4.292	0.0823	4.65	1.69
6B	22.2	7.3	493	10,941	242,891	162	3,598	9.872	6.614	0.0823	7.06	1.69
7A	19.5	5.8	380	7,415	144,590	113	2,205	7.858	4.236	0.0906	17.28	1.77
7B	19.5	5.9	380	7,415	144,590	115	2,243	7.858	3.834	0.0906	16.46	1.77
8A	27.4	11.7	751	20,571	563,641	321	8,784	14.655	8.732	0.0593	3.04	1.44
8B	27.4	12	751	20,571	563,641	329	9,009	14.655	7.049	0.0593	4.18	1.44
9A	32.8	20.4	1,076	35,288	1,157,432	669	21,947	20.884	0.234	0.1573	109.06	2.34
9B	32.8	20.9	1,076	35,288	1,157,432	686	22,485	20.884	0.000	0.1573	119.76	2.34
10A	26.5	20.9	702	18,610	493,155	554	14,677	13.742	51.239	0.0604	119.76	1.45
10B	26.5	21.3	702	18,610	493,155	564	14,958	13.742	57.125	0.0604	128.67	1.45
11A	33.5	22.3	1,122	37,595	1,259,445	747	25,026	21.785	0.265	0.1920	152.36	2.58
11B	33.5	22.6	1,122	37,595	1,259,445	757	25,363	21.785	0.664	0.1920	159.85	2.58
12A	28.1	18.4	790	22,188	623,484	517	14,529	15.390	9.060	0.0605	71.29	1.45
12B	28.1	17.8	790	22,188	623,484	500	14,055	15.390	5.808	0.0605	61.52	1.45
13A	9.6	3.7	92	885	8,493	36	341	3.223	0.227	0.0700	39.15	1.56
13B	9.6	3.9	92	885	8,493	37	359	3.223	0.458	0.0700	36.68	1.56
14A	5.3	4.2	28	149	789	22	118	2.556	2.704	0.1785	33.14	2.49
14B	5.3	4.3	28	149	789	23	121	2.556	3.043	0.1785	32.00	2.49
15A	18.1	6.8	328	5,930	107,328	123	2,228	6.940	0.020	0.0901	9.96	1.77
15B	18.1	7.1	328	5,930	107,328	129	2,326	6.940	0.025	0.0901	8.16	1.77

RESULTS OF POLYNOMIAL CORRELATION

Facility: Facility B
Location: Anytown USA

Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

Variable	Equation	Value
n	Number of data points =	30
S_1	S1 = Sum (xi) =	589
S_2	S2 = Sum (xi2) =	14,295
S_3	S3 = Sum (xi3) =	385,603
S_4	S4 = Sum (xi4) =	10,986,838
S_5	S5 = Sum (yi) =	299
S_6	S6 = Sum (xiyi) =	7,703
S_7	S7 = Sum (xi2yi) =	216,158
det A	det A = nS2S4-S2S2S2+S1S3S2-S3S3n+S2S1S3-S4S1S1 =	1.198E+10
b_0	b0 = (S5S2S4+S1S3S7+S2S6S3-S7S2S2-S3S3S5-S4S6S1)/det A =	2.854
b_1	b1 = (nS6S4+S5S3S2+S2S1S7-S2S6S2-S7S3n-S4S1S5)/det A =	-0.173
b_2	b2 = (nS2S7+S1S6S2+S5S1S3-S2S2S5-S3S6n-S7S1S1)/det A =	0.02203
S_p	S_p = Sqrt((1/(n-3)Sum of (y^-Y)2) =	2.873
D	D = n(S2S4-S3^2)+S1(S3S2-S1S4)+S2(S1S3-S2^2) =	1.198E+10
C_0	C0 = (S2S4-S3^2)/D =	0.698
C_1	C1 = (S3S2-S1S4)/D =	-0.0799
C_2	C2 = (S1S3-S2^2)/D =	1.895E-03
C_3	C3 = (nS4-S2^2)/D =	1.045E-02
C_4	C4 = (S1S2-nS3)/D =	-2.630E-04
C_5	C5 = (nS2-S1^2)/D =	6.855E-06
t_f	t_f from table =	2.052
EL	Emission Limit =	24.3
CI	CI = t_f*S_p *sqrt(delta-min) =	1.436
CI%	CI% = CI/EL*100=	5.91%
v_f	v_f from table =	1.293
$u_{n'}$	$u_{n'}$ from table =	1.181
n'	n' = 1/delta =	16.86
k_T	kt = un'*vf =	1.527
TI	TI = kt*sp =	4.385
TI%	TI% = TI/EL *100% =	18.0%
y~	y~ = 1/n*(Sum of (yi)) =	9.96
S_y	Sy = sqrt(Sum of (yi-y~)^2/(n-1) =	7.27
r^2	r^2 = 1-(S_p^2/Sy^2) =	0.844
r	r = sqrt((1-S_p^2/Sy^2)) =	0.919
Max-min?	b_2 > 0?	minimum
$x_{max-min}$	y = -b_1/2b_2 =	3.93
1.25x_{max}		41.88

* Indicates correlation coefficient is undefined.

Correlation equation: $y = 2.854 + -0.173\ x + 0.02203\ x^2$

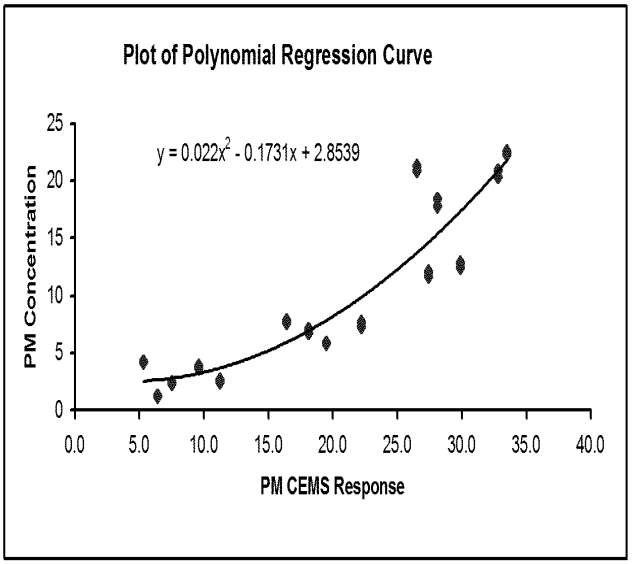
Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.919	≥ 0.85	yes
Confidence interval	5.91%	≤ 10%	yes
Tolerance interval	18.0%	≤ 25%	yes

* Indicates correlation coefficient is undefined.

Check for Correlation Curve Minimum/Maximum

Correlation curve minimum point	3.93
Minimum allowable x value	5.3
Is correlation curve minimum < minimum x value?	yes
Correlation curve maximum point	NA
Extrapolation limit for x (125% of maximum x value)	NA
Is correlation curve maximum > extrapolation limit?	NA



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR LOGARITHMIC CORRELATION

Facility: Facility B
Location: Anytown USA

Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

	PM CEMS response		PM concentration	CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	mg/acm					
Run	x	x' = ln (x)	y	$(x_i-\tilde{x})^2$	$(y_i-\tilde{y})^2$	$(x'_i-\tilde{x})(y_i-\tilde{y})$	y^	$(y^\wedge_i-y_i)^2$
1A	6.4	1.856	1.3	0.929	74.938	8.345	0.504	0.634
1B	6.4	1.856	1.2	0.929	76.679	8.441	0.504	0.485
2A	7.5	2.015	2.3	0.649	58.625	6.166	2.059	0.058
2B	7.5	2.015	2.5	0.649	55.602	6.005	2.059	0.194
3A	11.2	2.416	2.5	0.163	55.602	3.015	5.992	12.191
3B	11.2	2.416	2.8	0.163	51.218	2.894	5.992	10.186
4A	16.4	2.797	7.7	0.001	5.093	0.052	9.732	4.127
4B	16.4	2.797	7.9	0.001	4.230	0.047	9.732	3.355
5A	29.9	3.398	12.9	0.334	8.663	1.700	15.621	7.405
5B	29.9	3.398	12.5	0.334	6.469	1.469	15.621	9.742
6A	22.2	3.100	7.8	0.078	4.651	-0.604	12.701	24.021
6B	22.2	3.100	7.3	0.078	7.058	-0.743	12.701	29.172
7A	19.5	2.970	5.8	0.023	17.278	-0.624	11.429	31.690
7B	19.5	2.970	5.9	0.023	16.457	-0.609	11.429	30.574
8A	27.4	3.311	11.7	0.240	3.039	0.855	14.765	9.394
8B	27.4	3.311	12	0.240	4.175	1.002	14.765	7.645
9A	32.8	3.490	20.4	0.449	109.063	6.999	16.529	14.985
9B	32.8	3.490	20.9	0.449	119.757	7.334	16.529	19.106
10A	26.5	3.277	20.9	0.209	119.757	5.000	14.437	41.765
10B	26.5	3.277	21.3	0.209	128.671	5.183	14.437	47.095
11A	33.5	3.512	22.3	0.478	152.358	8.533	16.736	30.957
11B	33.5	3.512	22.6	0.478	159.854	8.740	16.736	34.386
12A	28.1	3.336	18.4	0.266	71.290	4.353	15.012	11.476
12B	28.1	3.336	17.8	0.266	61.518	4.043	15.012	7.771
13A	9.6	2.262	3.7	0.312	39.146	3.494	4.480	0.608
13B	9.6	2.262	3.9	0.312	36.683	3.382	4.480	0.336
14A	5.3	1.668	4.2	1.328	33.139	6.635	-1.346	30.756
14B	5.3	1.668	4.3	1.328	31.998	6.519	-1.346	31.875
15A	18.1	2.896	6.8	0.006	9.965	-0.239	10.699	15.200
15B	18.1	2.896	7.1	0.006	8.161	-0.216	10.699	12.951

RESULTS OF LOGARITHMIC CORRELATION

Facility: Facility B
Location: Anytown USA

Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

Variable	Equation	Value
n	Number of data points =	30
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.82
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	10.93
$y^{\sim}$	y~ = 1/n*(Sum of (y_i)) =	10.0
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	1,531
$S_{x'y}$	$S_{x'y}$ = Sum((x'_i-x'~)(y_i-y~)) =	107.2
b_0	b_0 = y~-b_1x'~ =	-17.700
b_1	b_1 = $S_{x'y}$/$S_{x'x'}$ =	9.807
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	4.141
$y^{\wedge}_{mean}$	y^ at mean x value =	9.957
t_f	t_f from table =	2.048
CI	CI = t_f*S_L*SQRT(1/n) =	1.549
EL	Emission Limit =	24.3
CI%	CI% =CI/EL*100% =	6.37%
n'	n' = n =	30
v_f	v_f from table =	1.286
$u_{n'}$	$u_{n'}$ from table =	1.168
k_T	k_t = u_n'*v_f =	1.503
TI	TI = k_t*SL =	6.22
TI%	TI% = TI/EL*100% =	25.6%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	7.266
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.675
r	r = SQRT((1-S_L^2/S_y^2)) =	0.822

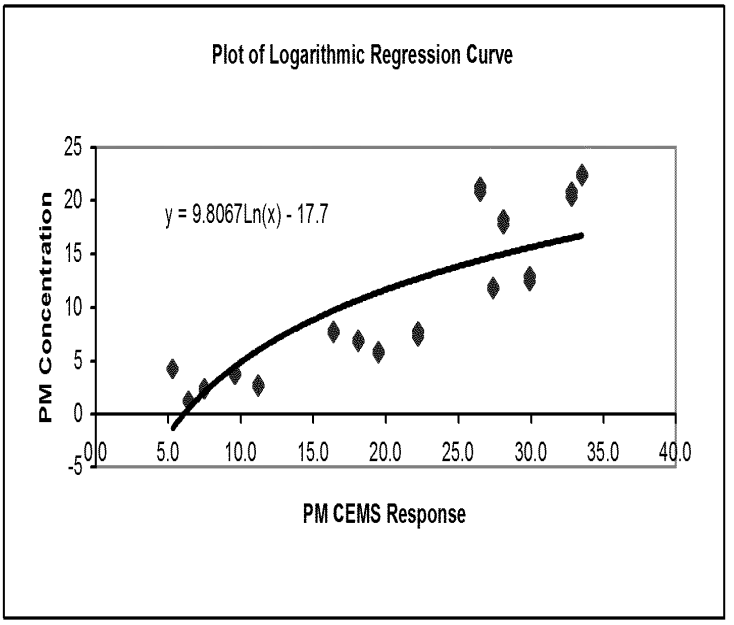
* Indicates correlation coefficient is undefined.

Correlation equation: y = -17.700 + 9.807 Ln(x)

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.822	≥ 0.85	no
Confidence interval	6.37%	≤ 10%	yes
Tolerance interval	25.6%	≤ 25%	no

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR EXPONENTIAL CORRELATION

Facility: Facility B
Location: Anytown USA

Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

Run	PM CEMS response x	PM concentration mg/acm Measured y	PM concentration mg/acm Transformed y' = Ln(y)	CALCULATED STATISTICAL PARAMETERS $(x_i-\bar{x})^2$	$(y'_i-\bar{y'})^2$	$(x_i-\bar{x})(y'_i-\bar{y'})$	$y'^{\wedge}$	$(y'^{\wedge}_i-y'_i)^2$
1A	6.4	1.3	0.262	174.94	2.957	22.744	0.886	0.389
1B	6.4	1.2	0.182	174.94	3.239	23.803	0.886	0.496
2A	7.5	2.3	0.833	147.06	1.320	13.934	0.978	0.021
2B	7.5	2.5	0.916	147.06	1.136	12.923	0.978	0.004
3A	11.2	2.5	0.916	71.01	1.136	8.980	1.284	0.135
3B	11.2	2.8	1.030	71.01	0.907	8.025	1.284	0.065
4A	16.4	7.7	2.041	10.41	0.004	-0.191	1.715	0.107
4B	16.4	7.9	2.067	10.41	0.007	-0.274	1.715	0.124
5A	29.9	12.9	2.557	105.54	0.331	5.910	2.833	0.076
5B	29.9	12.5	2.526	105.54	0.296	5.587	2.833	0.094
6A	22.2	7.8	2.054	6.62	0.005	0.186	2.195	0.020
6B	22.2	7.3	1.988	6.62	0.000	0.015	2.195	0.043
7A	19.5	5.8	1.758	0.02	0.050	0.028	1.971	0.046
7B	19.5	5.9	1.775	0.02	0.043	0.026	1.971	0.039
8A	27.4	11.7	2.460	60.42	0.228	3.713	2.626	0.028
8B	27.4	12	2.485	60.42	0.253	3.910	2.626	0.020
9A	32.8	20.4	3.016	173.54	1.068	13.616	3.073	0.003
9B	32.8	20.9	3.040	173.54	1.119	13.935	3.073	0.001
10A	26.5	20.9	3.040	47.24	1.119	7.271	2.551	0.239
10B	26.5	21.3	3.059	47.24	1.159	7.401	2.551	0.258
11A	33.5	22.3	3.105	192.47	1.260	15.575	3.131	0.001
11B	33.5	22.6	3.118	192.47	1.291	15.760	3.131	0.000
12A	28.1	18.4	2.912	71.80	0.866	7.884	2.684	0.052
12B	28.1	17.8	2.879	71.80	0.805	7.603	2.684	0.038
13A	9.6	3.7	1.308	100.53	0.454	6.754	1.151	0.025
13B	9.6	3.9	1.361	100.53	0.386	6.226	1.151	0.044
14A	5.3	4.2	1.435	205.25	0.299	7.835	0.795	0.409
14B	5.3	4.3	1.459	205.25	0.274	7.497	0.795	0.440
15A	18.1	6.8	1.917	2.33	0.004	0.099	1.855	0.004
15B	18.1	7.1	1.960	2.33	0.000	0.033	1.855	0.011

RESULTS OF EXPONENTIAL CORRELATION

Facility: Facility B
Location: Anytown USA

Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

Variable	Equation	Value
n	Number of data points =	30
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	19.63
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	2,738.4
$y'^{\sim}$	y'~ = 1/n*(Sum of (y'_i)) =	1.982
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	22.02
$S_{xy'}$	$S_{xy'}$ = Sum((x_i-x~)(y'_i-y'~)) =	226.81
b_0'	b_0' = y'~-b_1x~ =	0.356
b_0	b_0 = e^b_0' =	1.428
b_1	b_1 = $S_{xy'}$/S_{xx} =	0.083
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.340
$y'^{\wedge}_{mean}$	y'^ at mean x value =	1.982
t_f	t_f from table =	2.048
CI'	CI' = t_f*SL*SQRT(1/n) =	0.1270
LCL'	LCL' = y~' - CI'	1.855
UCL'	UCL' = y~' + CI'	2.109
CI	CI = (e^UCL' - e^LCL')/2	0.924
EL	Emission Limit =	24.3
CI%	CI% =CI/EL*100% =	3.80%
n'	n' = n =	30
v_f	v_f from table =	1.286
$u_{n'}$	$u_{n'}$ from table =	1.168
k_T	k_T = u_n'*v_f =	1.503
TI'	TI' = k_t*SL =	0.510
LTL'	LTL' = y~' - TI'	1.472
UTL'	UTL' = y~' + TI'	2.492
TI	TI = (e^UTL' - e^LTL')/2	3.866
TI%	TI% = TI/EL*100% =	15.9%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.871
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.848
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.921

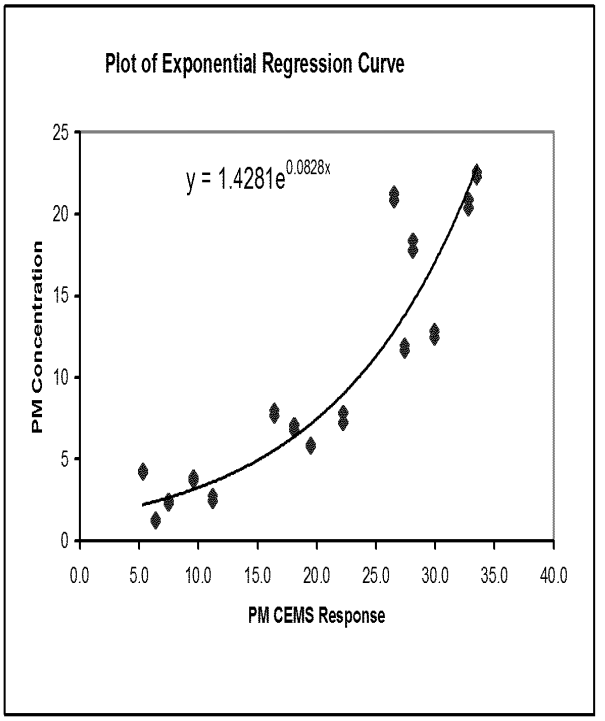
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 1.428\ e^{0.0828\ x}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.921	≥ 0.85	yes
Confidence interval	3.80%	≤ 10%	yes
Tolerance interval	15.9%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
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CALCULATIONS FOR POWER CORRELATION

Facility: Facility B
Location: Anytown USA

Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

	PM CEMS response		PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	Measured	Transformed					
Run	x	x' = Ln(x)	y	y' = Ln(y)	$(x'_i-x'^{\frown})^2$	$(y'_i-y'^{\frown})^2$	$(x'_i-x'^{\frown})(y'_i-y'^{\frown})$	y'^	$(y'_i^{\wedge}-y'_i)^2$
1A	6.4	1.856	1.3	0.262	0.929	2.957	1.658	0.756	0.2433
1B	6.4	1.856	1.2	0.182	0.929	3.239	1.735	0.756	0.3287
2A	7.5	2.015	2.3	0.833	0.649	1.320	0.925	0.957	0.0155
2B	7.5	2.015	2.5	0.916	0.649	1.136	0.858	0.957	0.0017
3A	11.2	2.416	2.5	0.916	0.163	1.136	0.431	1.468	0.3039
3B	11.2	2.416	2.8	1.030	0.163	0.907	0.385	1.468	0.1918
4A	16.4	2.797	7.7	2.041	0.001	0.004	-0.001	1.953	0.0078
4B	16.4	2.797	7.9	2.067	0.001	0.007	-0.002	1.953	0.0130
5A	29.9	3.398	12.9	2.557	0.334	0.331	0.332	2.717	0.0255
5B	29.9	3.398	12.5	2.526	0.334	0.296	0.314	2.717	0.0365
6A	22.2	3.100	7.8	2.054	0.078	0.005	0.020	2.338	0.0806
6B	22.2	3.100	7.3	1.988	0.078	0.000	0.002	2.338	0.1226
7A	19.5	2.970	5.8	1.758	0.023	0.050	-0.034	2.173	0.1723
7B	19.5	2.970	5.9	1.775	0.023	0.043	-0.031	2.173	0.1584
8A	27.4	3.311	11.7	2.460	0.240	0.228	0.234	2.606	0.0213
8B	27.4	3.311	12	2.485	0.240	0.253	0.247	2.606	0.0146
9A	32.8	3.490	20.4	3.016	0.449	1.068	0.693	2.835	0.0328
9B	32.8	3.490	20.9	3.040	0.449	1.119	0.709	2.835	0.0421
10A	26.5	3.277	20.9	3.040	0.209	1.119	0.483	2.563	0.2271
10B	26.5	3.277	21.3	3.059	0.209	1.159	0.492	2.563	0.2455
11A	33.5	3.512	22.3	3.105	0.478	1.260	0.776	2.861	0.0591
11B	33.5	3.512	22.6	3.118	0.478	1.291	0.785	2.861	0.0658
12A	28.1	3.336	18.4	2.912	0.266	0.866	0.480	2.638	0.0754
12B	28.1	3.336	17.8	2.879	0.266	0.805	0.463	2.638	0.0583
13A	9.6	2.262	3.7	1.308	0.312	0.454	0.376	1.271	0.0014
13B	9.6	2.262	3.9	1.361	0.312	0.386	0.347	1.271	0.0080
14A	5.3	1.668	4.2	1.435	1.328	0.299	0.630	0.516	0.8453
14B	5.3	1.668	4.3	1.459	1.328	0.274	0.603	0.516	0.8891
15A	18.1	2.896	6.8	1.917	0.006	0.004	-0.005	2.078	0.0260
15B	18.1	2.896	7.1	1.960	0.006	0.000	-0.002	2.078	0.0140

RESULTS OF POWER CORRELATION

Facility: Facility B
Location: Anytown USA

Emission Unit: Incinerator
Test Dates: April 3 to 6, 2003

Variable	Equation	Value
n	Number of data points =	30
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.820
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	10.93
$y'^{\sim}$	y'~ = 1/n*(Sum of (y'_i)) =	1.982
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	22.02
$S_{x'y'}$	$S_{x'y'}$ = Sum((x'_i-x'~)(y'_i-y'~)) =	13.90
b_0'	b_0' = y'~-b_1x~ =	-1.606
b_0	b_0 = e^b_0' =	0.201
b_1	b_1 = $S_{x'y'}$/$S_{x'x'}$ =	1.272
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.393
$y'^{\wedge}_{mean}$	y'^ at mean x value =	1.982
t_f	t_f from table =	2.048
CI'	CI = t_f*SL*SQRT(1/n) =	0.147
LCL'	LCL' = y~' - CI'	1.835
UCL'	UCL' = y~' + CI'	2.129
CI	CI = (e^UCL' - e^LCL')/2	1.071
EL	Emission Limit =	24.3
CI%	CI% =CI/EL*100% =	4.41%
n'	n' = n =	30
v_f	v_f from table =	1.286
$u_{n'}$	$u_{n'}$ from table =	1.168
k_T	k_T = $u_{n'}$*v_f =	1.503
TI'	TI' = $k_T S_L$ =	0.591
LTL'	LTL' = y~' - TI'	1.391
UTL'	UTL' = y~' + TI'	2.573
TI	TI = (e^UTL' - e^LTL')/2	4.540
TI%	TI% = TI/EL*100% =	18.7%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.871
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.796
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.892

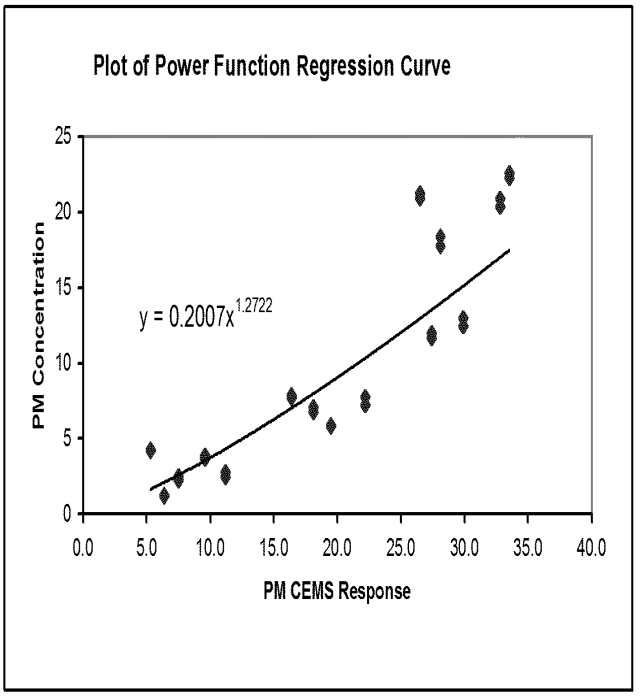
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 0.2007\ x^{1.2722}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.892	≥ 0.85	yes
Confidence interval	4.41%	≤ 10%	yes
Tolerance interval	18.7%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
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SELECTION OF BEST MODEL

Facility: **Facility B** **Emission Unit:** **Incinerator**
Location: **Anytown USA** **Test Dates:** **April 3 to 6, 2003**

Model	Correlation coefficient (a)	≥ 0.85 ?	Confidence interval half range percentage	≤ 10% ?	Tolerance interval half range percentage	≤ 25% ?	Min/max within allowable range?	Does model meet all criteria?
Linear	0.895	Yes	4.98%	Yes	20.0%	Yes	(b)	Yes
Polynomial	0.919	Yes	5.91%	Yes	18.0%	Yes	Yes	Yes
Logarithmic	0.822	No	6.37%	Yes	25.6%	No	(b)	No
Exponential	0.921	Yes	3.80%	Yes	15.9%	Yes	(b)	Yes
Power	0.892	Yes	4.41%	Yes	18.7%	Yes	(b)	Yes

(a) * indicates correlation coefficient is undefined; model does not satisfy criterion.
(b) Not applicable; criterion applies only to polynomial model.

Best model: **Exponential**

PS-11 Correlation Test
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Appendix A.3

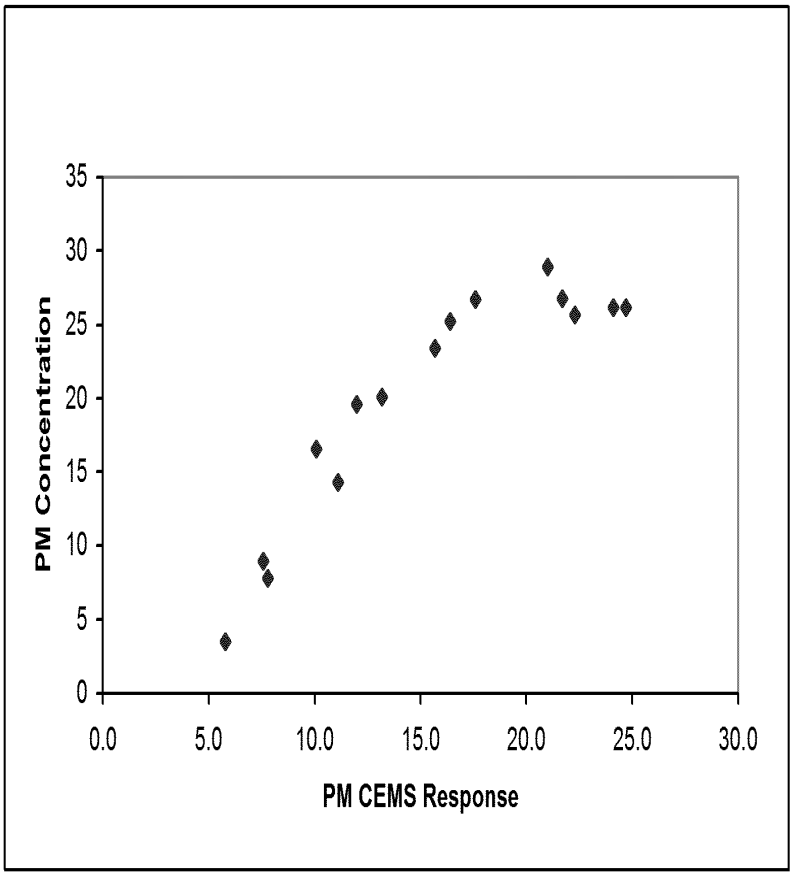
Complete Results of Example No. 3 – Initial Correlation Test

CORRELATION TEST PM CEMS AND REFERENCE METHOD TEST DATA

Run	PM CEMS response x	PM concentration mg/acm y
1	5.8	3.5
2	7.6	9
3	7.8	7.8
4	16.4	25.2
5	10.1	16.6
6	11.1	14.3
7	12.0	19.6
8	13.2	20.1
9	15.7	23.4
10	17.6	26.7
11	21.7	26.8
12	21.0	28.9
13	22.3	25.7
14	24.1	26.2
15	24.7	26.2

Facility: Facility C
Location: Anytown USA
Emission Unit: HWI
Test Dates: November 1 to 4, 2002

	Value	Units
Emission limit:	17.45	mg/acm



CALCULATIONS FOR LINEAR CORRELATION

Facility: Facility C
Location: Anytown USA
Emission Unit: HWI
Test Dates: November 1 to 4, 2002

Run	PM CEMS response x	PM concentration mg/acm y	CALCULATED STATISTICAL PARAMETERS				
			$(x_i-\tilde{x})^2$	$(y_i-\tilde{y})^2$	$(x_i-\tilde{x})(y_i-\tilde{y})$	$\hat{y}$	$(\hat{y}_i-y_i)^2$
1	5.8	3.5	92.29	272.25	158.51	8.942	29.621
2	7.6	9	60.94	121.00	85.87	11.014	4.058
3	7.8	7.8	57.86	148.84	92.80	11.245	11.865
4	16.4	25.2	0.99	27.04	5.17	21.143	16.456
5	10.1	16.6	28.16	11.56	18.04	13.892	7.334
6	11.1	14.3	18.55	32.49	24.55	15.043	0.552
7	12	19.6	11.61	0.16	1.36	16.079	12.399
8	13.2	20.1	4.87	0.01	-0.22	17.460	6.969
9	15.7	23.4	0.09	11.56	1.00	20.338	9.378
10	17.6	26.7	4.81	44.89	14.70	22.525	17.434
11	21.7	26.8	39.61	46.24	42.79	27.244	0.197
12	21	28.9	31.29	79.21	49.78	26.438	6.061
13	22.3	25.7	47.52	32.49	39.29	27.934	4.993
14	24.1	26.2	75.57	38.44	53.90	30.006	14.487
15	24.7	26.2	86.37	38.44	57.62	30.697	20.222

RESULTS OF LINEAR CORRELATION

Facility: Facility C
Location: Anytown USA

Emission Unit: HWI
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	15.41
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	561
$\tilde{y}$	y~ = 1/n*(Sum of (y_i)) =	20.00
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	905
S_{xy}	S_{xy} = Sum((x_i-x~)(y_i-y~)) =	645.2
b_0	b_0 = y~-b_1x~ =	2.267
b_1	b_1 = S_{xy}/S_{xx} =	1.151
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	3.530
$y^{\wedge}_{mean}$	y^ at mean x value =	20.000
t_f	t_f from table =	2.160
CI	CI = t_f*S_L*SQRT(1/n) =	1.969
EL	Emission Limit =	17.5
CI%	CI% =CI/EL*100% =	11.29%
n'	n' = n =	15
v_f	v_f from table =	1.485
$u_{n'}$	$u_{n'}$ from table =	1.184
k_T	k_t = u_n'*v_f =	1.759
TI	TI = k_t*S_L =	6.210
TI%	TI% = TI/EL*100% =	35.6%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	8.038
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.807
r	r = SQRT((1-S_L^2/S_y^2)) =	0.898

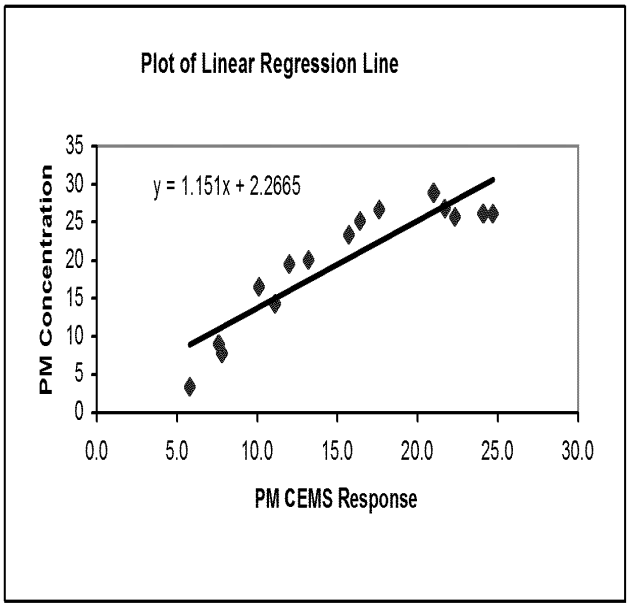
* Indicates correlation coefficient is undefined.

Correlation equation: y = 2.267 + 1.151 x

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.898	≥ 0.85	yes
Confidence interval	11.29%	≤ 10%	no
Tolerance interval	35.6%	≤ 25%	no

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR POLYNOMIAL CORRELATION

Facility: Facility C
Location: Anytown USA

Emission Unit: HWI
Test Dates: November 1 to 4, 2002

Run	PM CEMS response x	PM concentration mg/acm y	CALCULATED STATISTICAL PARAMETERS									
			x^2	x^3	x^4	xy	x^2y	y^	$(y^\wedge-y)^2$	delta	$(yi-y\sim)^2$	CI
1	5.8	3.5	34	195	1,132	20	118	3.138	0.131	0.4679	272.25	1.91
2	7.6	9	58	439	3,336	68	520	8.429	0.326	0.2223	121.00	1.31
3	7.8	7.8	61	475	3,702	61	475	8.976	1.384	0.2060	148.84	1.27
4	16.4	25.2	269	4,411	72,339	413	6,778	24.851	0.121	0.1650	27.04	1.13
5	10.1	16.6	102	1,030	10,406	168	1,693	14.689	3.653	0.1214	11.56	0.97
6	11.1	14.3	123	1,368	15,181	159	1,762	16.838	6.444	0.1224	32.49	0.98
7	12	19.6	144	1,728	20,736	235	2,822	18.600	1.000	0.1320	0.16	1.01
8	13.2	20.1	174	2,300	30,360	265	3,502	20.694	0.352	0.1488	0.01	1.08
9	15.7	23.4	246	3,870	60,757	367	5,768	24.119	0.517	0.1672	11.56	1.14
10	17.6	26.7	310	5,452	95,951	470	8,271	25.876	0.679	0.1541	44.89	1.09
11	21.7	26.8	471	10,218	221,737	582	12,620	27.177	0.142	0.1374	46.24	1.03
12	21	28.9	441	9,261	194,481	607	12,745	27.196	2.903	0.1265	79.21	0.99
13	22.3	25.7	497	11,090	247,297	573	12,780	27.082	1.911	0.1565	32.49	1.10
14	24.1	26.2	581	13,998	337,340	631	15,217	26.360	0.026	0.2949	38.44	1.51
15	24.7	26.2	610	15,069	372,210	647	15,984	25.973	0.051	0.3775	38.44	1.71

RESULTS OF POLYNOMIAL CORRELATION

Facility: Facility C
Location: Anytown USA

Emission Unit: HWI
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
S_1	S1 = Sum (xi) =	231
S_2	S2 = Sum (xi2) =	4,121
S_3	S3 = Sum (xi3) =	80,903
S_4	S4 = Sum (xi4) =	1,686,966
S_5	S5 = Sum (yi) =	300
S_6	S6 = Sum (xiyi) =	5,267
S_7	S7 = Sum (xi2yi) =	101,055
det A	det A = nS2S4-S2S2S2+S1S3S2-S3S3n+S2S1S3-S4S1S1 =	1.168E+08
b_0	b0 = (S5S2S4+S1S3S7+S2S6S3-S7S2S2-S3S3S5-S4S6S1)/det A =	-18.371
b_1	b1 = (nS6S4+S5S3S2+S2S1S7-S2S6S2-S7S3n-S4S1S5)/det A =	4.296
b_2	b2 = (nS2S7+S1S6S2+S5S1S3-S2S2S5-S3S6n-S7S1S1)/det A =	-0.10123
S_p	S_p = Sqrt((1/(n-3)Sum of (y^-Y)2) =	1.279
D	D = n(S2S4-S3^2)+S1(S3S2-S1S4)+S2(S1S3-S2^2) =	1.168E+08
C_0	C0 = (S2S4-S3^2)/D =	3.481
C_1	C1 = (S3S2-S1S4)/D =	-0.4833
C_2	C2 = (S1S3-S2^2)/D =	1.467E-02
C_3	C3 = (nS4-S2^2)/D =	7.124E-02
C_4	C4 = (S1S2-nS3)/D =	-2.236E-03
C_5	C5 = (nS2-S1^2)/D =	7.197E-05
t_f	t_f from table =	2.179
EL	Emission Limit =	17.5
CI	CI = t_f*S_p *sqrt(delta-min) =	0.971
CI%	CI% = CI/EL*100=	5.56%
v_f	v_f from table =	1.515
$u_{n'}$	$u_{n'}$ from table =	1.208
n'	n' = 1/delta =	8.24
k_T	kt = un'*vf =	1.831
TI	TI = kt*sp =	2.342
TI%	TI% = TI/EL *100% =	13.4%
y~	y~ = 1/n*(Sum of (yi)) =	20.00
S_y	Sy = sqrt(Sum of (yi-y~)^2/(n-1) =	8.04
r^2	r^2 = 1-(S_p^2/Sy^2) =	0.975
r	r = sqrt((1-S_p^2/Sy^2)) =	0.987
Max-min?	b_2 > 0?	maximum
$x_{max-min}$	y = -b_1/2b_2 =	21.22
1.25x_{max}		30.88

* Indicates correlation coefficient is undefined.

Correlation equation: $y = -18.371 + 4.296\ x + -0.10123\ x^2$

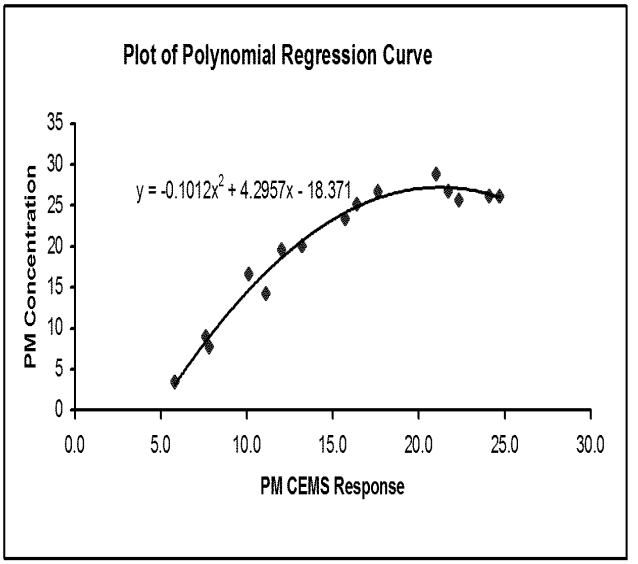
Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.987	≥ 0.85	yes
Confidence interval	5.56%	≤ 10%	yes
Tolerance interval	13.4%	≤ 25%	yes

* Indicates correlation coefficient is undefined.

Check for Correlation Curve Minimum/Maximum

Correlation curve minimum point	NA
Minimum allowable x value	NA
Is correlation curve minimum < minimum x value?	NA
Correlation curve maximum point	21.2
Extrapolation limit for x (125% of maximum x value)	30.9
Is correlation curve maximum > extrapolation limit?	no



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR LOGARITHMIC CORRELATION

Facility: Facility C
Location: Anytown USA

Emission Unit: HWI
Test Dates: November 1 to 4, 2002

	PM CEMS response		PM concentration	CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	mg/acm					
Run	x	x' = ln (x)	y	$(x_i-x'^{\sim})^2$	$(y_i-y^{\sim})^2$	$(x'_i-x'^{\sim})(y_i-y^{\sim})$	y^	$(y^{\wedge}_i-y_i)^2$
1	5.8	1.758	3.5	0.785	272.250	14.620	5.106	2.578
2	7.6	2.028	9	0.379	121.000	6.774	9.649	0.421
3	7.8	2.054	7.8	0.348	148.840	7.196	10.086	5.225
4	16.4	2.797	25.2	0.024	27.040	0.797	22.578	6.877
5	10.1	2.313	16.6	0.110	11.560	1.127	14.429	4.711
6	11.1	2.407	14.3	0.056	32.490	1.351	16.016	2.946
7	12	2.485	19.6	0.025	0.160	0.064	17.327	5.167
8	13.2	2.580	20.1	0.004	0.010	-0.006	18.929	1.371
9	15.7	2.754	23.4	0.012	11.560	0.373	21.844	2.420
10	17.6	2.868	26.7	0.050	44.890	1.501	23.765	8.616
11	21.7	3.077	26.8	0.188	46.240	2.947	27.285	0.235
12	21	3.045	28.9	0.160	79.210	3.565	26.734	4.693
13	22.3	3.105	25.7	0.212	32.490	2.626	27.743	4.175
14	24.1	3.182	26.2	0.290	38.440	3.337	29.048	8.111
15	24.7	3.207	26.2	0.317	38.440	3.490	29.461	10.637

RESULTS OF LOGARITHMIC CORRELATION

Facility: Facility C
Location: Anytown USA

Emission Unit: HWI
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.64
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	2.96
$y^{\sim}$	y~ = 1/n*(Sum of (y_i)) =	20.0
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	905
$S_{x'y}$	$S_{x'y}$ = Sum((x'_i-x'~)(y_i-y~)) =	49.8
b_0	b_0 = y~-b_1x'~ =	-24.443
b_1	b_1 = $S_{x'y}$/$S_{x'x'}$ =	16.809
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	2.290
$y^{\wedge}_{mean}$	y^ at mean x value =	20.000
t_f	t_f from table =	2.160
CI	CI = t_f*S_L*SQRT(1/n) =	1.277
EL	Emission Limit =	17.5
CI%	CI% =CI/EL*100% =	7.32%
n'	n' = n =	15
v_f	v_f from table =	1.485
$u_{n'}$	$u_{n'}$ from table =	1.184
k_T	k_t = u_n'*v_f =	1.759
TI	TI = k_t*SL =	4.03
TI%	TI% = TI/EL*100% =	23.1%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	8.038
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.919
r	r = SQRT((1-S_L^2/S_y^2)) =	0.959

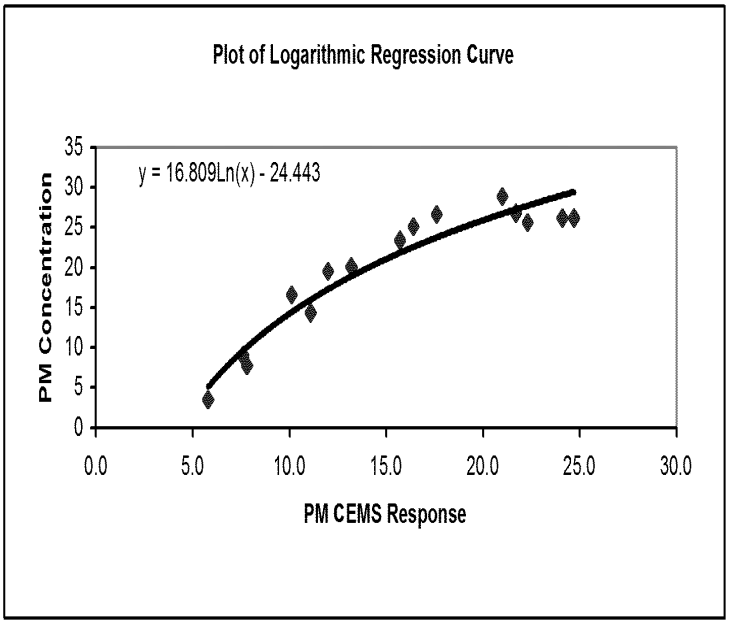
* Indicates correlation coefficient is undefined.

Correlation equation: y = -24.443 + 16.809 Ln(x)

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.959	≥ 0.85	yes
Confidence interval	7.32%	≤ 10%	yes
Tolerance interval	23.1%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR EXPONENTIAL CORRELATION

Facility: Facility C
Location: Anytown USA

Emission Unit: HWI
Test Dates: November 1 to 4, 2002

Run	PM CEMS response	PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
		Measured	Transformed					
	x	y	y' = Ln(y)	$(x_i-\bar{x})^2$	$(y'_i-\bar{y'})^2$	$(x_i-\bar{x})(y'_i-\bar{y'})$	y'^	$(y'^{\wedge}_i-y'_i)^2$
1	5.8	3.5	1.253	92.29	2.614	15.533	2.115	0.744
2	7.6	9	2.197	60.94	0.452	5.249	2.256	0.004
3	7.8	7.8	2.054	57.86	0.665	6.203	2.272	0.048
4	16.4	25.2	3.227	0.99	0.128	0.355	2.948	0.078
5	10.1	16.6	2.809	28.16	0.004	0.320	2.453	0.127
6	11.1	14.3	2.660	18.55	0.044	0.902	2.531	0.017
7	12	19.6	2.976	11.61	0.011	-0.361	2.602	0.139
8	13.2	20.1	3.001	4.87	0.017	-0.289	2.696	0.093
9	15.7	23.4	3.153	0.09	0.080	0.083	2.893	0.068
10	17.6	26.7	3.285	4.81	0.172	0.910	3.042	0.059
11	21.7	26.8	3.288	39.61	0.175	2.635	3.364	0.006
12	21	28.9	3.364	31.29	0.244	2.764	3.309	0.003
13	22.3	25.7	3.246	47.52	0.142	2.598	3.411	0.027
14	24.1	26.2	3.266	75.57	0.157	3.444	3.552	0.082
15	24.7	26.2	3.266	86.37	0.157	3.681	3.600	0.111

RESULTS OF EXPONENTIAL CORRELATION

Facility: Facility C
Location: Anytown USA

Emission Unit: HWI
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	15.41
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	560.5
$y'^{\sim}$	y'~ = 1/n*(Sum of (y'_i)) =	2.870
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	5.06
$S_{xy'}$	$S_{xy'}$ = Sum((x_i-x~)(y'_i-y'~)) =	44.03
b_0'	b_0' = y'~-b_1x~ =	1.659
b_0	b_0 = e^b_0' =	5.256
b_1	b_1 = $S_{xy'}$/S_{xx} =	0.079
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.351
$y'^{\wedge}_{mean}$	y'^ at mean x value =	2.870
t_f	t_f from table =	2.160
CI'	CI' = t_f*SL*SQRT(1/n) =	0.1960
LCL'	LCL' = y~' - CI'	2.674
UCL'	UCL' = y~' + CI'	3.066
CI	CI = (e^UCL' - e^LCL')/2	3.477
EL	Emission Limit =	17.5
CI%	CI% =CI/EL*100% =	19.93%
n'	n' = n =	15
v_f	v_f from table =	1.485
$u_{n'}$	$u_{n'}$ from table =	1.184
k_T	k_T = u_n'*v_f =	1.759
TI'	TI' = k_t*SL =	0.618
LTL'	LTL' = y~' - TI'	2.252
UTL'	UTL' = y~' + TI'	3.488
TI	TI = (e^UTL' - e^LTL')/2	11.602
TI%	TI% = TI/EL*100% =	66.5%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.601
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.659
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.812

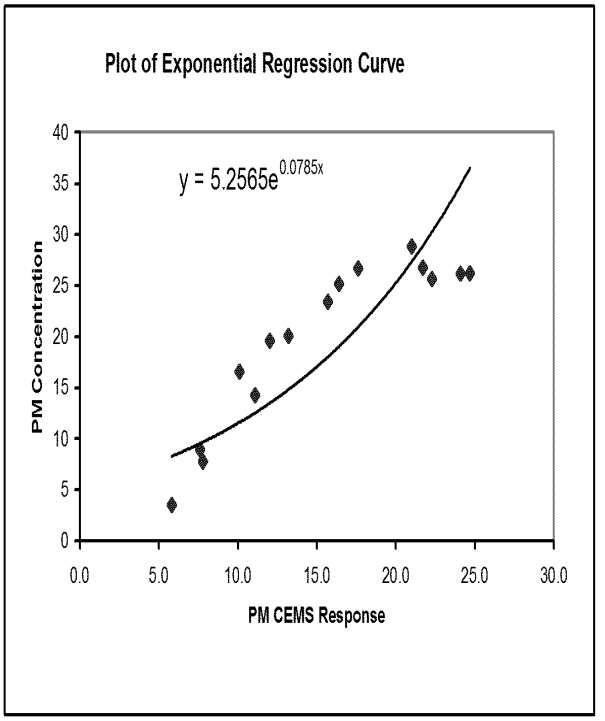
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 5.256\ e^{0.0785\ x}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.812	≥ 0.85	no
Confidence interval	19.93%	≤ 10%	no
Tolerance interval	66.5%	≤ 25%	no

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
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CALCULATIONS FOR POWER CORRELATION

Facility: Facility C
Location: Anytown USA

Emission Unit: HWI
Test Dates: November 1 to 4, 2002

	PM CEMS response		PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	Measured	Transformed					
Run	x	x' = Ln(x)	y	y' = Ln(y)	$(x'_i-x'^{\sim})^2$	$(y'_i-y'^{\sim})^2$	$(x'_i-x'^{\sim})(y'_i-y'^{\sim})$	y'^	$(y'_i{}^{\wedge}-y'_i)^2$
1	5.8	1.758	3.5	1.253	0.785	2.614	1.433	1.810	0.3105
2	7.6	2.028	9	2.197	0.379	0.452	0.414	2.133	0.0041
3	7.8	2.054	7.8	2.054	0.348	0.665	0.481	2.164	0.0121
4	16.4	2.797	25.2	3.227	0.024	0.128	0.055	3.053	0.0302
5	10.1	2.313	16.6	2.809	0.110	0.004	0.020	2.473	0.1129
6	11.1	2.407	14.3	2.660	0.056	0.044	0.050	2.586	0.0055
7	12	2.485	19.6	2.976	0.025	0.011	-0.017	2.679	0.0877
8	13.2	2.580	20.1	3.001	0.004	0.017	-0.008	2.793	0.0430
9	15.7	2.754	23.4	3.153	0.012	0.080	0.031	3.001	0.0231
10	17.6	2.868	26.7	3.285	0.050	0.172	0.093	3.137	0.0217
11	21.7	3.077	26.8	3.288	0.188	0.175	0.181	3.388	0.0099
12	21	3.045	28.9	3.364	0.160	0.244	0.198	3.349	0.0002
13	22.3	3.105	25.7	3.246	0.212	0.142	0.174	3.421	0.0303
14	24.1	3.182	26.2	3.266	0.290	0.157	0.213	3.513	0.0613
15	24.7	3.207	26.2	3.266	0.317	0.157	0.223	3.543	0.0767

RESULTS OF POWER CORRELATION

Facility: Facility C
Location: Anytown USA

Emission Unit: HWI
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
$x'^{\sim}$	$x'\sim = 1/n*(\text{Sum of } (x'_i)) =$	2.644
$S_{x'x'}$	$S_{x'x'} = \text{Sum}((x'_i-x'\sim)^2) =$	2.96
$y'^{\sim}$	$y'\sim = 1/n*(\text{Sum of } (y'_i)) =$	2.870
$S_{y'y'}$	$S_{y'y'} = \text{Sum}((y'_i-y'\sim)^2) =$	5.06
$S_{x'y'}$	$S_{x'y'} = \text{Sum}((x'_i-x'\sim)(y'_i-y'\sim)) =$	3.54
b_0'	$b_0' = y'\sim-b_1x\sim =$	-0.292
b_0	$b_0 = e^{\wedge}b_0' =$	0.747
b_1	$b_1 = S_{x'y'}/S_{x'x'} =$	1.196
S_L	$S_L = \text{SQRT}(1/(n-2)(\text{Sum}(y'^{\wedge}_i-y'_i)^2)) =$	0.253
$y'^{\wedge}_{mean}$	$y'^{\wedge}$ at mean x value =	2.870
t_f	t_f from table =	2.160
CI'	$CI = t_f*SL*\text{SQRT}(1/n) =$	0.141
LCL'	$LCL' = y\sim' - CI'$	2.729
UCL'	$UCL' = y\sim' + CI'$	3.011
CI	$CI = (e^{\wedge}UCL' - e^{\wedge}LCL')/2$	2.492
EL	Emission Limit =	17.5
CI%	$CI\% = CI/EL*100\% =$	14.28%
n'	$n' = n =$	15
v_f	v_f from table =	1.485
$u_{n'}$	$u_{n'}$ from table =	1.184
k_T	$k_T = u_{n'}*v_f =$	1.759
TI'	$TI' = k_TS_L =$	0.444
LTL'	$LTL' = y\sim' - TI'$	2.425
UTL'	$UTL' = y\sim' + TI'$	3.314
TI	$TI = (e^{\wedge}UTL' - e^{\wedge}LTL')/2$	8.093
TI%	$TI\% = TI/EL*100\% =$	46.4%
$S_{y'}$	$S_{y'} = \text{SQRT}(S_{y'y'}/(n-1)) =$	0.601
r^2	$r^2 = 1-(SL^{\wedge}2/S_{y'}^{\wedge}2) =$	0.824
r	$r = \text{SQRT}((1-SL^{\wedge}2/S_{y'}^{\wedge}2)) =$	0.908

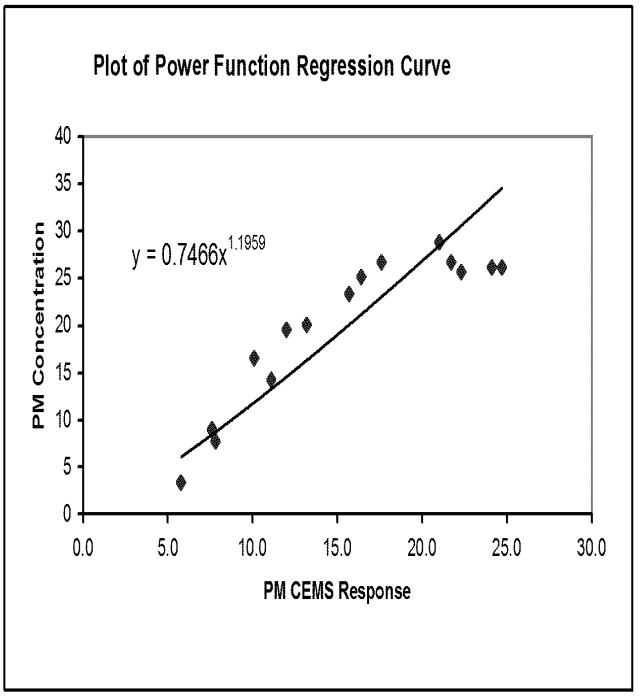
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 0.7466\ x^{1.1959}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.908	≥ 0.85	yes
Confidence interval	14.28%	$\leq 10\%$	no
Tolerance interval	46.4%	$\leq 25\%$	no

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
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SELECTION OF BEST MODEL

Facility: **Facility C** **Emission Unit:** **HWI**
Location: **Anytown USA** **Test Dates:** **November 1 to 4, 2002**

Model	Correlation coefficient (a)	≥ 0.85 ?	Confidence interval half range percentage	≤ 10% ?	Tolerance interval half range percentage	≤ 25% ?	Min/max within allowable range?	Does model meet all criteria?
Linear	0.898	Yes	11.29%	No	35.6%	No	(b)	No
Polynomial	0.987	Yes	5.56%	Yes	13.4%	Yes	No	No
Logarithmic	0.959	Yes	7.32%	Yes	23.1%	Yes	(b)	Yes
Exponential	0.812	No	19.93%	No	66.5%	No	(b)	No
Power	0.908	Yes	14.28%	No	46.4%	No	(b)	No

(a) * indicates correlation coefficient is undefined; model does not satisfy criterion.
(b) Not applicable; criterion applies only to polynomial model.

Best model: **Logarithmic**

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Appendix A.4

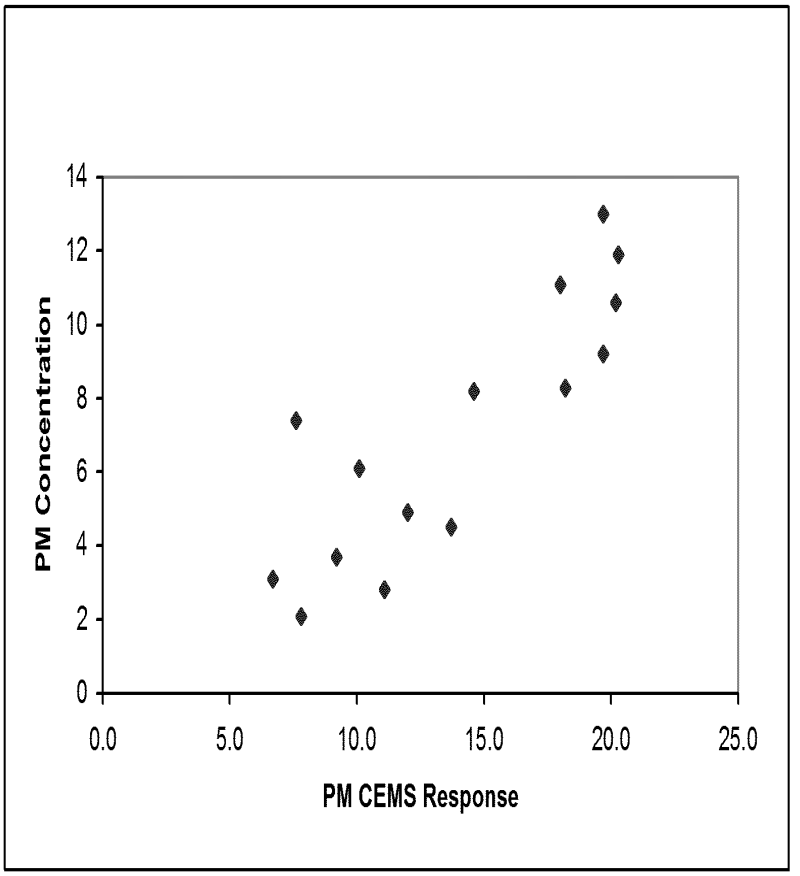
Complete Results of Example No. 4
Original Test Data

CORRELATION TEST PM CEMS AND REFERENCE METHOD TEST DATA

Run	PM CEMS response x	PM concentration mg/acm y
1	6.7	3.1
2	7.6	7.4
3	7.8	2.1
4	9.2	3.7
5	10.1	6.1
6	11.1	2.8
7	12.0	4.9
8	18.2	8.3
9	13.7	4.5
10	14.6	8.2
11	19.7	13.0
12	18.0	11.1
13	20.3	11.9
14	20.2	10.6
15	19.7	9.2

Facility: Facility D
Location: Anytown USA
Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

	Value	Units
Emission limit:	18.5	mg/acm



CALCULATIONS FOR LINEAR CORRELATION

Facility: Facility D
Location: Anytown USA
Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

Run	PM CEMS response x	PM concentration mg/acm y	CALCULATED STATISTICAL PARAMETERS				
			$(x_i-\tilde{x})^2$	$(y_i-\tilde{y})^2$	$(x_i-\tilde{x})(y_i-\tilde{y})$	$\hat{y}$	$(\hat{y}_i-y_i)^2$
1	6.7	3.1	52.22	16.21	29.10	2.808	0.085
2	7.6	7.4	40.03	0.07	-1.73	3.346	16.437
3	7.8	2.1	37.54	25.27	30.80	3.465	1.864
4	9.2	3.7	22.34	11.74	16.20	4.302	0.362
5	10.1	6.1	14.64	1.05	3.93	4.840	1.588
6	11.1	2.8	7.99	18.72	12.23	5.437	6.956
7	12	4.9	3.71	4.96	4.29	5.975	1.156
8	18.2	8.3	18.26	1.38	5.01	9.680	1.906
9	13.7	4.5	0.05	6.90	0.60	6.991	6.206
10	14.6	8.2	0.45	1.15	0.72	7.529	0.450
11	19.7	13	33.33	34.50	33.91	10.577	5.872
12	18	11.1	16.59	15.79	16.18	9.561	2.369
13	20.3	11.9	40.62	22.78	30.42	10.935	0.930
14	20.2	10.6	39.35	12.06	21.79	10.876	0.076
15	19.7	9.2	33.33	4.30	11.97	10.577	1.896

RESULTS OF LINEAR CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	13.93
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	360
$\tilde{y}$	y~ = 1/n*(Sum of (y_i)) =	7.13
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	177
S_{xy}	S_{xy} = Sum((x_i-x~)(y_i-y~)) =	215.4
b_0	b_0 = y~-b_1x~ =	-1.196
b_1	b_1 = S_{xy}/S_{xx} =	0.598
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	1.925
$y^{\wedge}_{mean}$	y^ at mean x value =	7.127
t_f	t_f from table =	2.160
CI	CI = t_f*S_L*SQRT(1/n) =	1.074
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	5.80%
n'	n' = n =	15
v_f	v_f from table =	1.485
$u_{n'}$	$u_{n'}$ from table =	1.184
k_T	k_t = u_n'*v_f =	1.759
TI	TI = k_t*S_L =	3.386
TI%	TI% = TI/EL*100% =	18.3%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	3.555
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.707
r	r = SQRT((1-S_L^2/S_y^2)) =	0.841

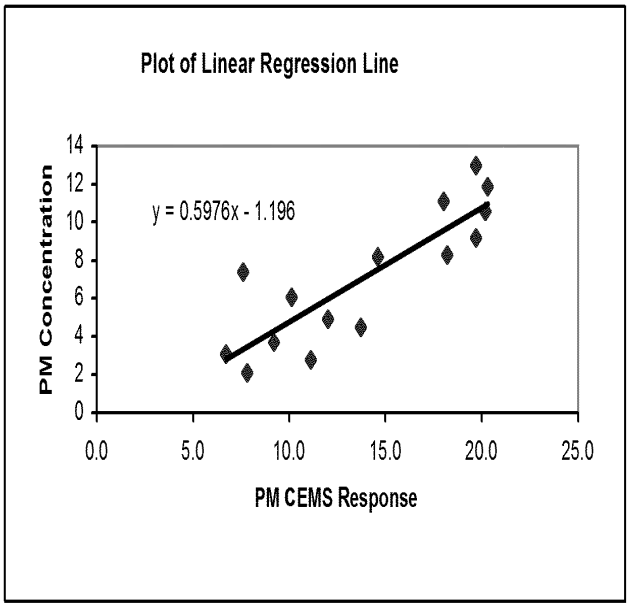
* Indicates correlation coefficient is undefined.

Correlation equation: y = -1.196 + 0.598 x

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.841	≥ 0.85	no
Confidence interval	5.80%	≤ 10%	yes
Tolerance interval	18.3%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



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CALCULATIONS FOR POLYNOMIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

	PM CEMS response	PM concentration mg/acm	CALCULATED STATISTICAL PARAMETERS									
Run	x	y	x^2	x^3	x^4	xy	x^2y	y^	$(y^\wedge-y)^2$	delta	$(yi-y\sim)^2$	CI
1	6.7	3.1	45	301	2,015	21	139	4.024	0.854	0.4111	16.21	2.57
2	7.6	7.4	58	439	3,336	56	427	4.032	11.346	0.2412	0.07	1.97
3	7.8	2.1	61	475	3,702	16	128	4.043	3.775	0.2158	25.27	1.87
4	9.2	3.7	85	779	7,164	34	313	4.219	0.270	0.1296	11.74	1.45
5	10.1	6.1	102	1,030	10,406	62	622	4.423	2.812	0.1307	1.05	1.45
6	11.1	2.8	123	1,368	15,181	31	345	4.733	3.735	0.1559	18.72	1.59
7	12	4.9	144	1,728	20,736	59	706	5.085	0.034	0.1838	4.96	1.72
8	18.2	8.3	331	6,029	109,720	151	2,749	9.437	1.292	0.1253	1.38	1.42
9	13.7	4.5	188	2,571	35,228	62	845	5.945	2.088	0.2145	6.90	1.86
10	14.6	8.2	213	3,112	45,437	120	1,748	6.502	2.883	0.2103	1.15	1.84
11	19.7	13	388	7,645	150,614	256	5,045	10.993	4.027	0.1825	34.50	1.72
12	18	11.1	324	5,832	104,976	200	3,596	9.244	3.444	0.1262	15.79	1.43
13	20.3	11.9	412	8,365	169,818	242	4,904	11.671	0.053	0.2523	22.78	2.02
14	20.2	10.6	408	8,242	166,497	214	4,325	11.556	0.913	0.2382	12.06	1.96
15	19.7	9.2	388	7,645	150,614	181	3,570	10.993	3.216	0.1825	4.30	1.72

RESULTS OF POLYNOMIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
S_1	S1 = Sum (xi) =	209
S_2	S2 = Sum (xi2) =	3,270
S_3	S3 = Sum (xi3) =	55,562
S_4	S4 = Sum (xi4) =	995,443
S_5	S5 = Sum (yi) =	107
S_6	S6 = Sum (xiyi) =	1,704
S_7	S7 = Sum (xi2yi) =	29,463
det A	det A = nS2S4-S2S2S2+S1S3S2-S3S3n+S2S1S3-S4S1S1 =	2.107E+07
b_0	b0 = (S5S2S4+S1S3S7+S2S6S3-S7S2S2-S3S3S5-S4S6S1)/det A =	6.189
b_1	b1 = (nS6S4+S5S3S2+S2S1S7-S2S6S2-S7S3n-S4S1S5)/det A =	-0.615
b_2	b2 = (nS2S7+S1S6S2+S5S1S3-S2S2S5-S3S6n-S7S1S1)/det A =	0.04361
S_p	S_p = Sqrt((1/(n-3)Sum of (y^-Y)2) =	1.843
D	D = n(S2S4-S3^2)+S1(S3S2-S1S4)+S2(S1S3-S2^2) =	2.107E+07
C_0	C0 = (S2S4-S3^2)/D =	7.963
C_1	C1 = (S3S2-S1S4)/D =	-1.2473
C_2	C2 = (S1S3-S2^2)/D =	4.346E-02
C_3	C3 = (nS4-S2^2)/D =	2.013E-01
C_4	C4 = (S1S2-nS3)/D =	-7.138E-03
C_5	C5 = (nS2-S1^2)/D =	2.567E-04
t_f	t_f from table =	2.179
EL	Emission Limit =	18.5
CI	CI = t_f*S_p *sqrt(delta-min) =	1.421
CI%	CI% = CI/EL*100=	7.68%
v_f	v_f from table =	1.515
$u_{n'}$	$u_{n'}$ from table =	1.208
n'	n' = 1/delta =	7.98
k_T	kt = un'*vf =	1.831
TI	TI = kt*sp =	3.373
TI%	TI% = TI/EL *100% =	18.2%
y~	y~ = 1/n*(Sum of (yi)) =	7.13
S_y	Sy = sqrt(Sum of (yi-y~)^2/(n-1) =	3.55
r^2	r^2 = 1-(S_p^2/Sy^2) =	0.731
r	r = sqrt((1-S_p^2/Sy^2)) =	0.855
Max-min?	b_2 > 0?	minimum
$x_{max-min}$	y = -b_1/2b_2 =	7.05
1.25x_{max}		25.38

* Indicates correlation coefficient is undefined.

Correlation equation: $y = 6.189 + -0.615\ x + 0.04361\ x^2$

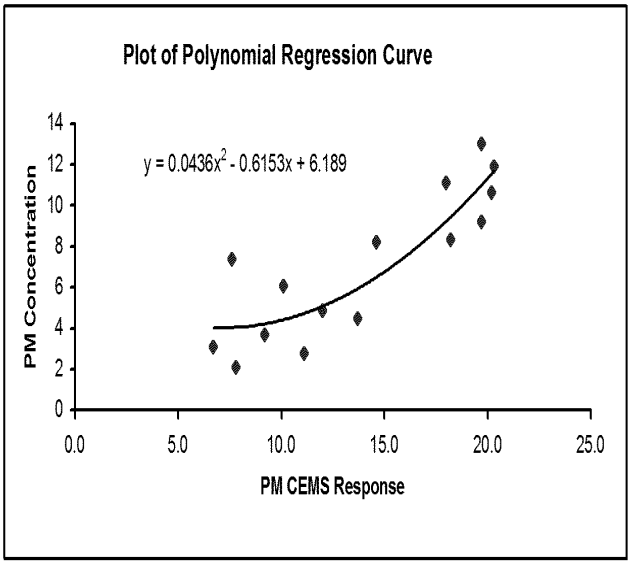
Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.855	≥ 0.85	yes
Confidence interval	7.68%	≤ 10%	yes
Tolerance interval	18.2%	≤ 25%	yes

* Indicates correlation coefficient is undefined.

Check for Correlation Curve Minimum/Maximum

Correlation curve minimum point	7.05
Minimum allowable x value	6.7
Is correlation curve minimum < minimum x value?	no
Correlation curve maximum point	NA
Extrapolation limit for x (125% of maximum x value)	NA
Is correlation curve maximum > extrapolation limit?	NA



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR LOGARITHMIC CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

	PM CEMS response		PM concentration					
	Measured	Transformed	mg/acm	CALCULATED STATISTICAL PARAMETERS				
Run	x	x' = ln (x)	y	$(x_i-x'^{\sim})^2$	$(y_i-y^{\sim})^2$	$(x'_i-x'^{\sim})(y_i-y^{\sim})$	y^	$(y^{\wedge}_i-y_i)^2$
1	6.7	1.902	3.1	0.439	16.214	2.669	2.244	0.732
2	7.6	2.028	7.4	0.288	0.075	-0.147	3.173	17.871
3	7.8	2.054	2.1	0.261	25.267	2.568	3.364	1.597
4	9.2	2.219	3.7	0.120	11.742	1.185	4.580	0.774
5	10.1	2.313	6.1	0.064	1.054	0.259	5.267	0.694
6	11.1	2.407	2.8	0.025	18.720	0.684	5.962	10.000
7	12	2.485	4.9	0.006	4.958	0.178	6.536	2.678
8	18.2	2.901	8.3	0.113	1.377	0.395	9.604	1.700
9	13.7	2.617	4.5	0.003	6.899	-0.138	7.512	9.073
10	14.6	2.681	8.2	0.013	1.152	0.124	7.981	0.048
11	19.7	2.981	13	0.173	34.496	2.441	10.187	7.912
12	18	2.890	11.1	0.106	15.787	1.293	9.523	2.488
13	20.3	3.011	11.9	0.199	22.785	2.127	10.408	2.226
14	20.2	3.006	10.6	0.194	12.064	1.530	10.372	0.052
15	19.7	2.981	9.2	0.173	4.299	0.862	10.187	0.975

RESULTS OF LOGARITHMIC CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.57
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	2.18
$y^{\sim}$	y~ = 1/n*(Sum of (y_i)) =	7.1
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	177
$S_{x'y}$	$S_{x'y}$ = Sum((x'_i-x'~)(y_i-y~)) =	16.0
b_0	b_0 = y~-b_1x'~ =	-11.764
b_1	b_1 = $S_{x'y}$/$S_{x'x'}$ =	7.365
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	2.127
$y^{\wedge}_{mean}$	y^ at mean x value =	7.127
t_f	t_f from table =	2.160
CI	CI = t_f*S_L*SQRT(1/n) =	1.187
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	6.41%
n'	n' = n =	15
v_f	v_f from table =	1.485
$u_{n'}$	$u_{n'}$ from table =	1.184
k_T	k_t = u_n'*v_f =	1.759
TI	TI = k_t*SL =	3.74
TI%	TI% = TI/EL*100% =	20.2%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	3.555
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.642
r	r = SQRT((1-S_L^2/S_y^2)) =	0.801

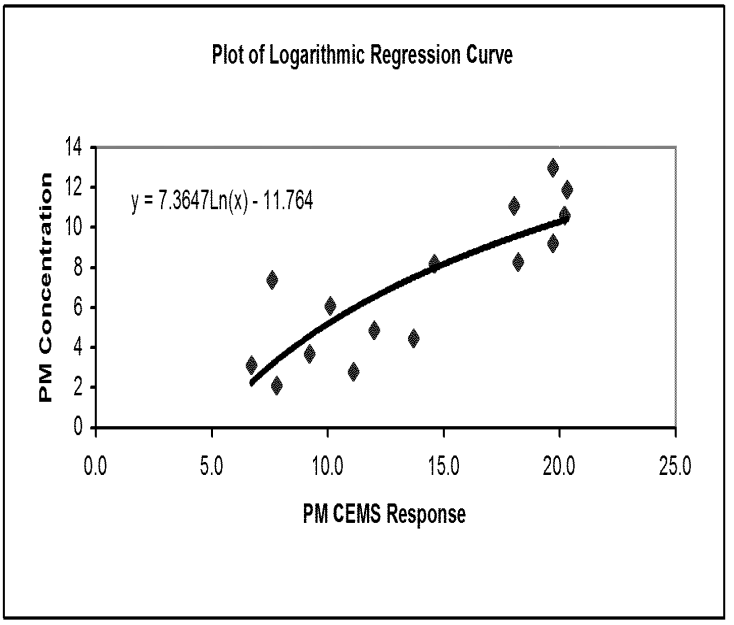
* Indicates correlation coefficient is undefined.

Correlation equation: y = -11.764 + 7.365 Ln(x)

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.801	≥ 0.85	no
Confidence interval	6.41%	≤ 10%	yes
Tolerance interval	20.2%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
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CALCULATIONS FOR EXPONENTIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

Run	PM CEMS response	PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
		Measured	Transformed					
	x	y	y' = Ln(y)	$(x_i-\bar{x})^2$	$(y'_i-\bar{y'})^2$	$(x_i-\bar{x})(y'_i-\bar{y'})$	y'^	$(y'^{\wedge}_i-y'_i)^2$
1	6.7	3.1	1.131	52.22	0.480	5.007	1.149	0.000
2	7.6	7.4	2.001	40.03	0.031	-1.121	1.233	0.590
3	7.8	2.1	0.742	37.54	1.171	6.631	1.252	0.260
4	9.2	3.7	1.308	22.34	0.266	2.438	1.383	0.006
5	10.1	6.1	1.808	14.64	0.000	0.061	1.467	0.117
6	11.1	2.8	1.030	7.99	0.631	2.246	1.560	0.281
7	12	4.9	1.589	3.71	0.055	0.453	1.644	0.003
8	18.2	8.3	2.116	18.26	0.085	1.248	2.223	0.011
9	13.7	4.5	1.504	0.05	0.102	0.073	1.803	0.089
10	14.6	8.2	2.104	0.45	0.078	0.188	1.887	0.047
11	19.7	13	2.565	33.33	0.549	4.276	2.364	0.041
12	18	11.1	2.407	16.59	0.340	2.374	2.205	0.041
13	20.3	11.9	2.477	40.62	0.426	4.157	2.420	0.003
14	20.2	10.6	2.361	39.35	0.288	3.367	2.410	0.002
15	19.7	9.2	2.219	33.33	0.156	2.280	2.364	0.021

RESULTS OF EXPONENTIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	13.93
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	360.5
$\tilde{y'}$	y'~ = 1/n*(Sum of (y'_i)) =	1.824
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	4.66
$S_{xy'}$	$S_{xy'}$ = Sum((x_i-x~)(y'_i-y'~)) =	33.68
b_0'	b_0' = y'~-b_1x~ =	0.523
b_0	b_0 = e^b_0' =	1.687
b_1	b_1 = $S_{xy'}$/S_{xx} =	0.093
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.341
$y'^{\wedge}_{mean}$	y'^ at mean x value =	1.824
t_f	t_f from table =	2.160
CI'	CI' = t_f*SL*SQRT(1/n) =	0.1903
LCL'	LCL' = y~' - CI'	1.634
UCL'	UCL' = y~' + CI'	2.015
CI	CI = (e^UCL' - e^LCL')/2	1.187
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	6.41%
n'	n' = n =	15
v_f	v_f from table =	1.485
$u_{n'}$	$u_{n'}$ from table =	1.184
k_T	k_T = u_n'*v_f =	1.759
TI'	TI' = k_t*SL =	0.600
LTL'	LTL' = y~' - TI'	1.224
UTL'	UTL' = y~' + TI'	2.424
TI	TI = (e^UTL' - e^LTL')/2	3.947
TI%	TI% = TI/EL*100% =	21.3%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.577
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.650
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.806

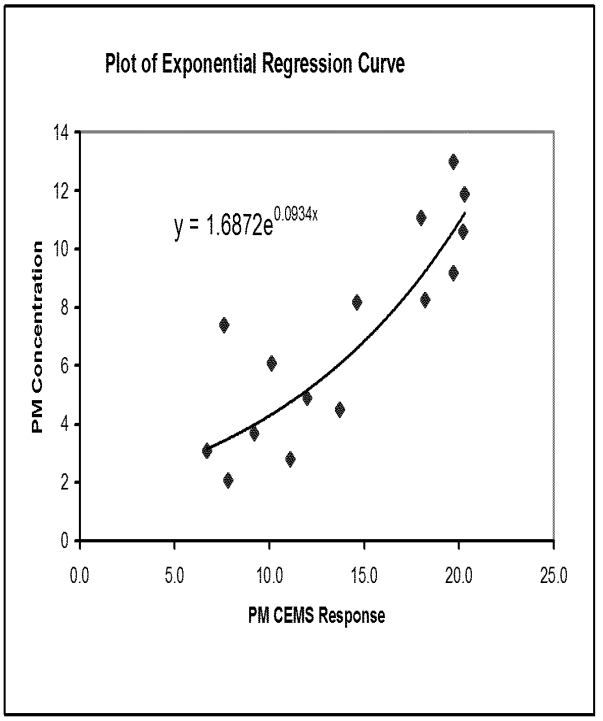
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 1.687\ e^{0.0934\ x}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.806	≥ 0.85	no
Confidence interval	6.41%	≤ 10%	yes
Tolerance interval	21.3%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
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CALCULATIONS FOR POWER CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

	PM CEMS response		PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	Measured	Transformed					
Run	x	x' = Ln(x)	y	y' = Ln(y)	$(x'_i-x'^{\sim})^2$	$(y'_i-y'^{\sim})^2$	$(x'_i-x'^{\sim})(y'_i-y'^{\sim})$	y'^	$(y'_i{}^{\wedge}-y'_i)^2$
1	6.7	1.902	3.1	1.131	0.439	0.480	0.459	1.049	0.0068
2	7.6	2.028	7.4	2.001	0.288	0.031	-0.095	1.196	0.6486
3	7.8	2.054	2.1	0.742	0.261	1.171	0.553	1.227	0.2348
4	9.2	2.219	3.7	1.308	0.120	0.266	0.178	1.420	0.0124
5	10.1	2.313	6.1	1.808	0.064	0.000	0.004	1.529	0.0781
6	11.1	2.407	2.8	1.030	0.025	0.631	0.126	1.639	0.3717
7	12	2.485	4.9	1.589	0.006	0.055	0.019	1.730	0.0199
8	18.2	2.901	8.3	2.116	0.113	0.085	0.098	2.218	0.0103
9	13.7	2.617	4.5	1.504	0.003	0.102	-0.017	1.885	0.1454
10	14.6	2.681	8.2	2.104	0.013	0.078	0.032	1.960	0.0208
11	19.7	2.981	13	2.565	0.173	0.549	0.308	2.310	0.0648
12	18	2.890	11.1	2.407	0.106	0.340	0.190	2.205	0.0409
13	20.3	3.011	11.9	2.477	0.199	0.426	0.291	2.345	0.0172
14	20.2	3.006	10.6	2.361	0.194	0.288	0.236	2.340	0.0004
15	19.7	2.981	9.2	2.219	0.173	0.156	0.164	2.310	0.0083

RESULTS OF POWER CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: November 1 to 4, 2002

Variable	Equation	Value
n	Number of data points =	15
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.565
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	2.18
$y'^{\sim}$	y'~ = 1/n*(Sum of (y'_i)) =	1.824
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	4.66
$S_{x'y'}$	$S_{x'y'}$ = Sum((x'_i-x'~)(y'_i-y'~)) =	2.55
b_0'	b_0' = y'~-b_1x~ =	-1.177
b_0	b_0 = e^b_0' =	0.308
b_1	b_1 = $S_{x'y'}$/$S_{x'x'}$ =	1.170
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.360
$y'^{\wedge}_{mean}$	y'^ at mean x value =	1.824
t_f	t_f from table =	2.160
CI'	CI = t_f*SL*SQRT(1/n) =	0.201
LCL'	LCL' = y~' - CI'	1.624
UCL'	UCL' = y~' + CI'	2.025
CI	CI = (e^UCL' - e^LCL')/2	1.251
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	6.76%
n'	n' = n =	15
v_f	v_f from table =	1.485
$u_{n'}$	$u_{n'}$ from table =	1.184
k_T	k_T = $u_{n'}$*v_f =	1.759
TI'	TI' = $k_T S_L$ =	0.632
LTL'	LTL' = y~' - TI'	1.192
UTL'	UTL' = y~' + TI'	2.457
TI	TI = (e^UTL' - e^LTL')/2	4.187
TI%	TI% = TI/EL*100% =	22.6%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.577
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.612
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.782

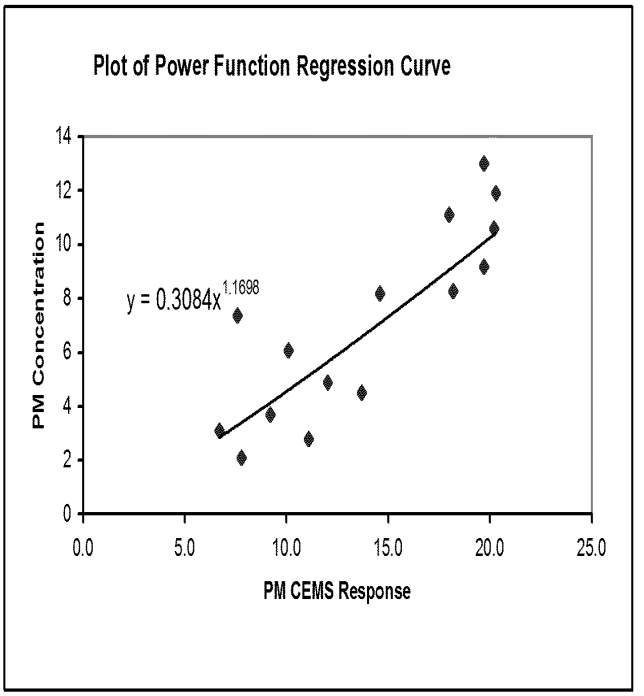
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 0.3084\ x^{1.1698}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.782	≥ 0.85	no
Confidence interval	6.76%	≤ 10%	yes
Tolerance interval	22.6%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
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SELECTION OF BEST MODEL

Facility: **Facility D** **Emission Unit:** **Boiler**
Location: **Anytown USA** **Test Dates:** **November 1 to 4, 2002**

Model	Correlation coefficient (a)	≥ 0.85 ?	Confidence interval half range percentage	≤ 10% ?	Tolerance interval half range percentage	≤ 25% ?	Min/max within allowable range?	Does model meet all criteria?
Linear	0.841	No	5.80%	Yes	18.3%	Yes	(b)	No
Polynomial	0.855	Yes	7.68%	Yes	18.2%	Yes	No	No
Logarithmic	0.801	No	6.41%	Yes	20.2%	Yes	(b)	No
Exponential	0.806	No	6.41%	Yes	21.3%	Yes	(b)	No
Power	0.782	No	6.76%	Yes	22.6%	Yes	(b)	No

(a) * indicates correlation coefficient is undefined; model does not satisfy criterion.
(b) Not applicable; criterion applies only to polynomial model.

Best model: **None acceptable**

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Appendix A.5

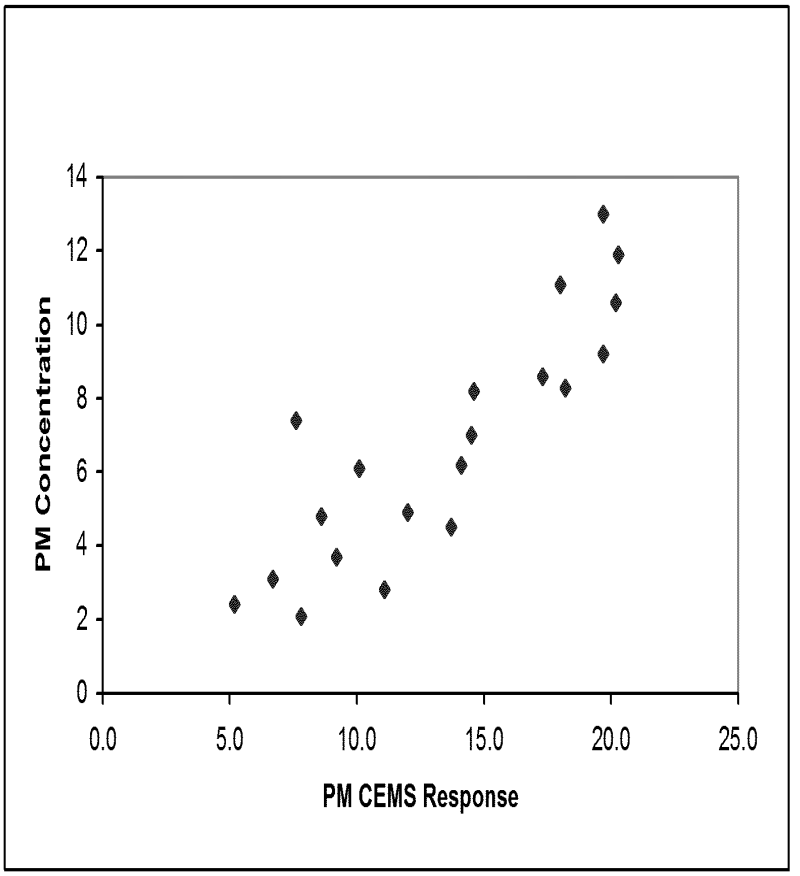
Complete Results of Example No. 4 With Five Additional Test Runs

CORRELATION TEST PM CEMS AND REFERENCE METHOD TEST DATA

Run	PM CEMS response x	PM concentration mg/acm y
1	6.7	3.1
2	7.6	7.4
3	7.8	2.1
4	9.2	3.7
5	10.1	6.1
6	11.1	2.8
7	12.0	4.9
8	18.2	8.3
9	13.7	4.5
10	14.6	8.2
11	19.7	13.0
12	18.0	11.1
13	20.3	11.9
14	20.2	10.6
15	19.7	9.2
16	5.2	2.4
17	8.6	4.8
18	14.1	6.2
19	14.5	7.0
20	17.3	8.6

Facility: Facility D
Location: Anytown USA
Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

	Value	Units
Emission limit:	18.5	mg/acm



CALCULATIONS FOR LINEAR CORRELATION

Facility: Facility D
Location: Anytown USA
Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Run	PM CEMS response x	PM concentration mg/acm y	CALCULATED STATISTICAL PARAMETERS				
			$(x_i-\tilde{x})^2$	$(y_i-\tilde{y})^2$	$(x_i-\tilde{x})(y_i-\tilde{y})$	$\hat{y}$	$(\hat{y}_i-y_i)^2$
1	6.7	3.1	45.29	13.65	24.87	2.936	0.027
2	7.6	7.4	33.99	0.37	-3.53	3.452	15.583
3	7.8	2.1	31.70	22.04	26.43	3.567	2.152
4	9.2	3.7	17.89	9.58	13.09	4.370	0.449
5	10.1	6.1	11.09	0.48	2.31	4.886	1.474
6	11.1	2.8	5.43	15.96	9.31	5.459	7.071
7	12	4.9	2.04	3.59	2.71	5.975	1.156
8	18.2	8.3	22.75	2.27	7.18	9.530	1.512
9	13.7	4.5	0.07	5.27	-0.62	6.950	6.002
10	14.6	8.2	1.37	1.97	1.64	7.466	0.539
11	19.7	13	39.31	38.50	38.91	10.390	6.813
12	18	11.1	20.88	18.53	19.67	9.415	2.839
13	20.3	11.9	47.20	26.06	35.07	10.734	1.360
14	20.2	10.6	45.83	14.48	25.76	10.677	0.006
15	19.7	9.2	39.31	5.78	15.08	10.390	1.416
16	5.2	2.4	67.73	19.32	36.17	2.076	0.105
17	8.6	4.8	23.33	3.98	9.64	4.026	0.599
18	14.1	6.2	0.45	0.35	-0.40	7.179	0.959
19	14.5	7	1.14	0.04	0.22	7.408	0.167
20	17.3	8.6	14.98	3.26	6.99	9.014	0.171

RESULTS OF LINEAR CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	20
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	13.43
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	472
$\tilde{y}$	y~ = 1/n*(Sum of (y_i)) =	6.80
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	205
S_{xy}	S_{xy} = Sum((x_i-x~)(y_i-y~)) =	270.5
b_0	b_0 = y~-b_1x~ =	-0.905
b_1	b_1 = S_{xy}/S_{xx} =	0.573
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	1.673
$y^{\wedge}_{mean}$	y^ at mean x value =	6.795
t_f	t_f from table =	2.101
CI	CI = t_f*S_L*SQRT(1/n) =	0.786
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	4.25%
n'	n' = n =	20
v_f	v_f from table =	1.384
$u_{n'}$	$u_{n'}$ from table =	1.177
k_T	k_t = u_n'*v_f =	1.629
TI	TI = k_t*S_L =	2.726
TI%	TI% = TI/EL*100% =	14.7%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	3.289
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.741
r	r = SQRT((1-S_L^2/S_y^2)) =	0.861

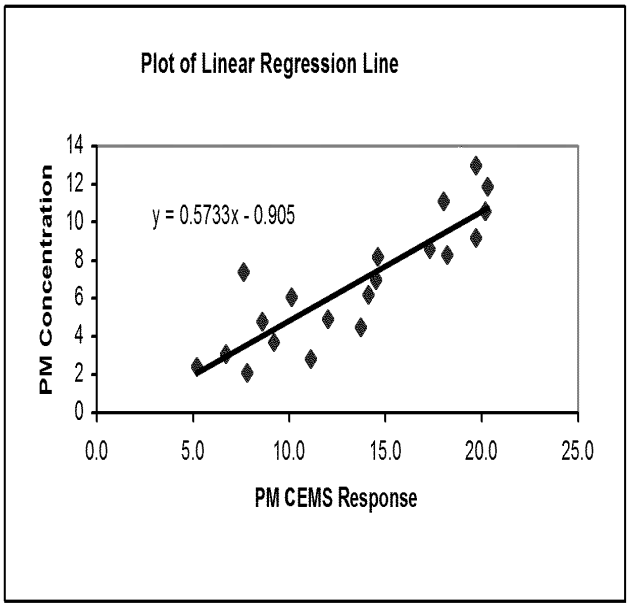
* Indicates correlation coefficient is undefined.

Correlation equation: y = -0.905 + 0.573 x

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.861	≥ 0.85	yes
Confidence interval	4.25%	≤ 10%	yes
Tolerance interval	14.7%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR POLYNOMIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

	PM CEMS response	PM concentration mg/acm	CALCULATED STATISTICAL PARAMETERS									
Run	x	y	x^2	x^3	x^4	xy	x^2y	y^	$(y^\wedge-y)^2$	delta	$(yi-y\sim)^2$	CI
1	6.7	3.1	45	301	2,015	21	139	3.578	0.229	0.2035	13.65	1.52
2	7.6	7.4	58	439	3,336	56	427	3.745	13.360	0.1340	0.37	1.23
3	7.8	2.1	61	475	3,702	16	128	3.789	2.852	0.1240	22.04	1.19
4	9.2	3.7	85	779	7,164	34	313	4.167	0.218	0.0937	9.58	1.03
5	10.1	6.1	102	1,030	10,406	62	622	4.474	2.643	0.0972	0.48	1.05
6	11.1	2.8	123	1,368	15,181	31	345	4.876	4.310	0.1090	15.96	1.11
7	12	4.9	144	1,728	20,736	59	706	5.291	0.153	0.1197	3.59	1.16
8	18.2	8.3	331	6,029	109,720	151	2,749	9.531	1.516	0.0982	2.27	1.05
9	13.7	4.5	188	2,571	35,228	62	845	6.214	2.937	0.1259	5.27	1.19
10	14.6	8.2	213	3,112	45,437	120	1,748	6.776	2.029	0.1195	1.97	1.16
11	19.7	13	388	7,645	150,614	256	5,045	10.919	4.329	0.1725	38.50	1.40
12	18	11.1	324	5,832	104,976	200	3,596	9.357	3.039	0.0947	18.53	1.04
13	20.3	11.9	412	8,365	169,818	242	4,904	11.514	0.149	0.2351	26.06	1.63
14	20.2	10.6	408	8,242	166,497	214	4,325	11.413	0.662	0.2230	14.48	1.59
15	19.7	9.2	388	7,645	150,614	181	3,570	10.919	2.956	0.1725	5.78	1.40
16	5.2	2.4	27	141	731	12	65	3.413	1.027	0.4433	19.32	2.24
17	8.6	4.8	74	636	5,470	41	355	3.990	0.657	0.0996	3.98	1.06
18	14.1	6.2	199	2,803	39,525	87	1,233	6.457	0.066	0.1238	0.35	1.18
19	14.5	7	210	3,049	44,205	102	1,472	6.711	0.084	0.1205	0.04	1.17
20	17.3	8.6	299	5,178	89,575	149	2,574	8.766	0.028	0.0903	3.26	1.01

RESULTS OF POLYNOMIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	20
S_1	S1 = Sum (xi) =	269
S_2	S2 = Sum (xi2) =	4,079
S_3	S3 = Sum (xi3) =	67,368
S_4	S4 = Sum (xi4) =	1,174,949
S_5	S5 = Sum (yi) =	136
S_6	S6 = Sum (xiyi) =	2,096
S_7	S7 = Sum (xi2yi) =	35,161
det A	det A = nS2S4-S2S2S2+S1S3S2-S3S3n+S2S1S3-S4S1S1 =	6.865E+07
b_0	b0 = (S5S2S4+S1S3S7+S2S6S3-S7S2S2-S3S3S5-S4S6S1)/det A =	3.935
b_1	b1 = (nS6S4+S5S3S2+S2S1S7-S2S6S2-S7S3n-S4S1S5)/det A =	-0.263
b_2	b2 = (nS2S7+S1S6S2+S5S1S3-S2S2S5-S3S6n-S7S1S1)/det A =	0.03137
S_p	S_p = Sqrt((1/(n-3)Sum of (y^-Y)2) =	1.595
D	D = n(S2S4-S3^2)+S1(S3S2-S1S4)+S2(S1S3-S2^2) =	6.865E+07
C_0	C0 = (S2S4-S3^2)/D =	3.705
C_1	C1 = (S3S2-S1S4)/D =	-0.5942
C_2	C2 = (S1S3-S2^2)/D =	2.121E-02
C_3	C3 = (nS4-S2^2)/D =	9.993E-02
C_4	C4 = (S1S2-nS3)/D =	-3.667E-03
C_5	C5 = (nS2-S1^2)/D =	1.375E-04
t_f	t_f from table =	2.110
EL	Emission Limit =	18.5
CI	CI = t_f*S_p *sqrt(delta-min) =	1.011
CI%	CI% = CI/EL*100=	5.47%
v_f	v_f from table =	1.400
$u_{n'}$	$u_{n'}$ from table =	1.195
n'	n' = 1/delta =	11.07
k_T	kt = un'*vf =	1.673
TI	TI = kt*sp =	2.668
TI%	TI% = TI/EL *100% =	14.4%
y~	y~ = 1/n*(Sum of (yi)) =	6.80
S_y	Sy = sqrt(Sum of (yi-y~)^2/(n-1) =	3.29
r^2	r^2 = 1-(S_p^2/Sy^2) =	0.765
r	r = sqrt((1-S_p^2/Sy^2)) =	0.875
Max-min?	b_2 > 0?	minimum
$x_{max-min}$	y = -b_1/2b_2 =	4.20
1.25x_{max}		25.38

* Indicates correlation coefficient is undefined.

Correlation equation: $y = 3.935 + -0.263\ x + 0.03137\ x^2$

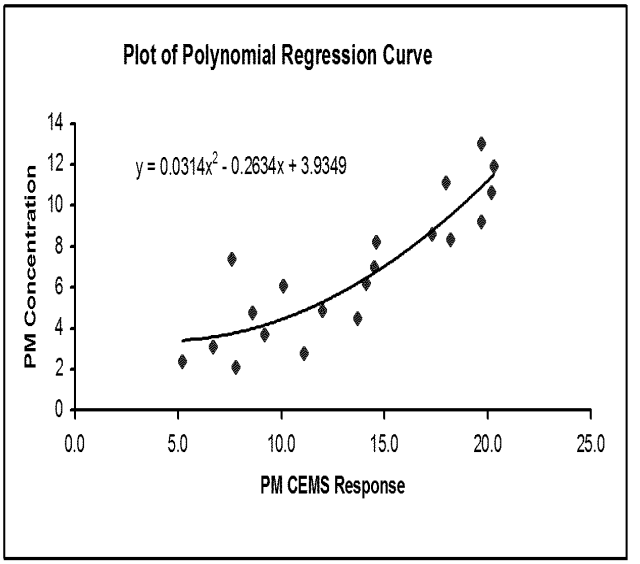
Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.875	≥ 0.85	yes
Confidence interval	5.47%	≤ 10%	yes
Tolerance interval	14.4%	≤ 25%	yes

* Indicates correlation coefficient is undefined.

Check for Correlation Curve Minimum/Maximum

Correlation curve minimum point	4.20
Minimum allowable x value	5.2
Is correlation curve minimum < minimum x value?	yes
Correlation curve maximum point	NA
Extrapolation limit for x (125% of maximum x value)	NA
Is correlation curve maximum > extrapolation limit?	NA



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR LOGARITHMIC CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

	PM CEMS response		PM concentration					
	Measured	Transformed	mg/acm	CALCULATED STATISTICAL PARAMETERS				
Run	x	x' = ln (x)	y	$(x_i-\bar{x'})^2$	$(y_i-\bar{y})^2$	$(x'_i-\bar{x'})(y_i-\bar{y})$	y^	$(\hat{y}_i-y_i)^2$
1	6.7	1.902	3.1	0.385	13.653	2.292	2.717	0.146
2	7.6	2.028	7.4	0.244	0.366	-0.299	3.546	14.854
3	7.8	2.054	2.1	0.219	22.043	2.198	3.717	2.614
4	9.2	2.219	3.7	0.092	9.579	0.938	4.802	1.214
5	10.1	2.313	6.1	0.044	0.483	0.146	5.416	0.468
6	11.1	2.407	2.8	0.013	15.960	0.461	6.036	10.473
7	12	2.485	4.9	0.001	3.591	0.071	6.549	2.718
8	18.2	2.901	8.3	0.144	2.265	0.570	9.287	0.974
9	13.7	2.617	4.5	0.009	5.267	-0.218	7.420	8.525
10	14.6	2.681	8.2	0.025	1.974	0.223	7.838	0.131
11	19.7	2.981	13	0.210	38.502	2.844	9.808	10.190
12	18	2.890	11.1	0.135	18.533	1.584	9.214	3.555
13	20.3	3.011	11.9	0.238	26.061	2.493	10.005	3.591
14	20.2	3.006	10.6	0.234	14.478	1.839	9.973	0.394
15	19.7	2.981	9.2	0.210	5.784	1.102	9.808	0.369
16	5.2	1.649	2.4	0.763	19.316	3.840	1.051	1.820
17	8.6	2.152	4.8	0.137	3.980	0.739	4.359	0.195
18	14.1	2.646	6.2	0.015	0.354	-0.074	7.609	1.985
19	14.5	2.674	7	0.023	0.042	0.031	7.793	0.629
20	17.3	2.851	8.6	0.108	3.258	0.593	8.954	0.125

RESULTS OF LOGARITHMIC CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	20
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.52
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	3.25
$y^{\sim}$	y~ = 1/n*(Sum of (y_i)) =	6.8
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	205
$S_{x'y}$	$S_{x'y}$ = Sum((x'_i-x'~)(y_i-y~)) =	21.4
b_0	b_0 = y~-b_1x'~ =	-9.788
b_1	b_1 = $S_{x'y}$/$S_{x'x'}$ =	6.574
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	1.900
$y^{\wedge}_{mean}$	y^ at mean x value =	6.795
t_f	t_f from table =	2.101
CI	CI = t_f*S_L*SQRT(1/n) =	0.893
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	4.82%
n'	n' = n =	20
v_f	v_f from table =	1.384
$u_{n'}$	$u_{n'}$ from table =	1.177
k_T	k_t = u_n'*v_f =	1.629
TI	TI = k_t*SL =	3.09
TI%	TI% = TI/EL*100% =	16.7%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	3.289
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.666
r	r = SQRT((1-S_L^2/S_y^2)) =	0.816

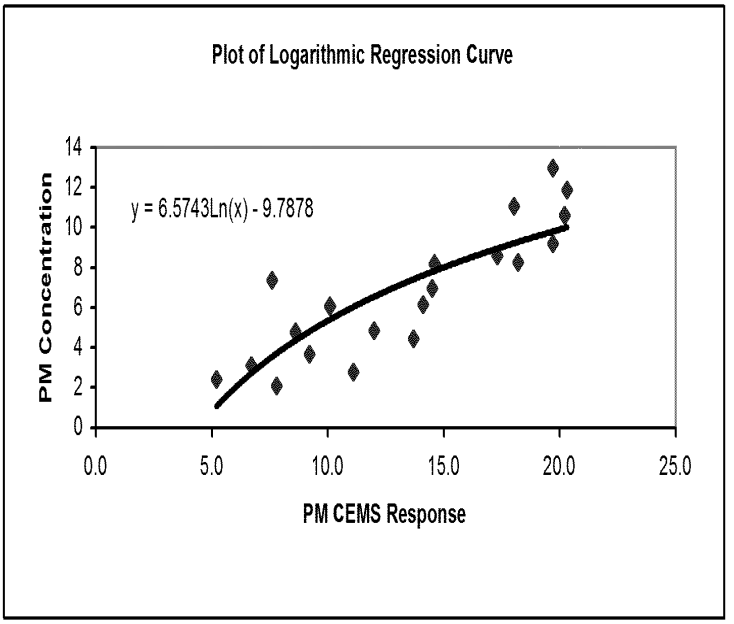
* Indicates correlation coefficient is undefined.

Correlation equation: y = -9.788 + 6.574 Ln(x)

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.816	≥ 0.85	no
Confidence interval	4.82%	≤ 10%	yes
Tolerance interval	16.7%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR EXPONENTIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Run	PM CEMS response	PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
		Measured	Transformed					
	x	y	y' = Ln(y)	$(x_i-\bar{x})^2$	$(y'_i-\bar{y'})^2$	$(x_i-\bar{x})(y'_i-\bar{y'})$	y'^	$(y'^{\wedge}_i-y'_i)^2$
1	6.7	3.1	1.131	45.29	0.429	4.409	1.157	0.001
2	7.6	7.4	2.001	33.99	0.046	-1.253	1.241	0.578
3	7.8	2.1	0.742	31.70	1.091	5.881	1.260	0.268
4	9.2	3.7	1.308	17.89	0.229	2.023	1.391	0.007
5	10.1	6.1	1.808	11.09	0.000	-0.073	1.475	0.111
6	11.1	2.8	1.030	5.43	0.573	1.763	1.569	0.290
7	12	4.9	1.589	2.04	0.039	0.282	1.653	0.004
8	18.2	8.3	2.116	22.75	0.109	1.573	2.233	0.014
9	13.7	4.5	1.504	0.07	0.080	-0.076	1.812	0.095
10	14.6	8.2	2.104	1.37	0.101	0.372	1.896	0.043
11	19.7	13	2.565	39.31	0.606	4.881	2.373	0.037
12	18	11.1	2.407	20.88	0.385	2.836	2.214	0.037
13	20.3	11.9	2.477	47.20	0.476	4.741	2.429	0.002
14	20.2	10.6	2.361	45.83	0.330	3.889	2.420	0.003
15	19.7	9.2	2.219	39.31	0.187	2.713	2.373	0.024
16	5.2	2.4	0.875	67.73	0.830	7.498	1.017	0.020
17	8.6	4.8	1.569	23.33	0.047	1.052	1.335	0.055
18	14.1	6.2	1.825	0.45	0.001	0.026	1.849	0.001
19	14.5	7	1.946	1.14	0.025	0.171	1.887	0.004
20	17.3	8.6	2.152	14.98	0.133	1.414	2.148	0.000

RESULTS OF EXPONENTIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	20
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	13.43
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	471.8
$y'^{\sim}$	y'~ = 1/n*(Sum of (y'_i)) =	1.786
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	5.72
$S_{xy'}$	$S_{xy'}$ = Sum((x_i-x~)(y'_i-y'~)) =	44.12
b_0'	b_0' = y'~-b_1x~ =	0.531
b_0	b_0 = e^b_0' =	1.700
b_1	b_1 = $S_{xy'}$/S_{xx} =	0.094
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.298
$y'^{\wedge}_{mean}$	y'^ at mean x value =	1.786
t_f	t_f from table =	2.101
CI'	CI' = t_f*SL*SQRT(1/n) =	0.1398
LCL'	LCL' = y~' - CI'	1.647
UCL'	UCL' = y~' + CI'	1.926
CI	CI = (e^UCL' - e^LCL')/2	0.837
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	4.52%
n'	n' = n =	20
v_f	v_f from table =	1.384
$u_{n'}$	$u_{n'}$ from table =	1.177
k_T	k_T = u_n'*v_f =	1.629
TI'	TI' = k_t*SL =	0.485
LTL'	LTL' = y~' - TI'	1.302
UTL'	UTL' = y~' + TI'	2.271
TI	TI = (e^UTL' - e^LTL')/2	3.007
TI%	TI% = TI/EL*100% =	16.3%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.549
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.706
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.840

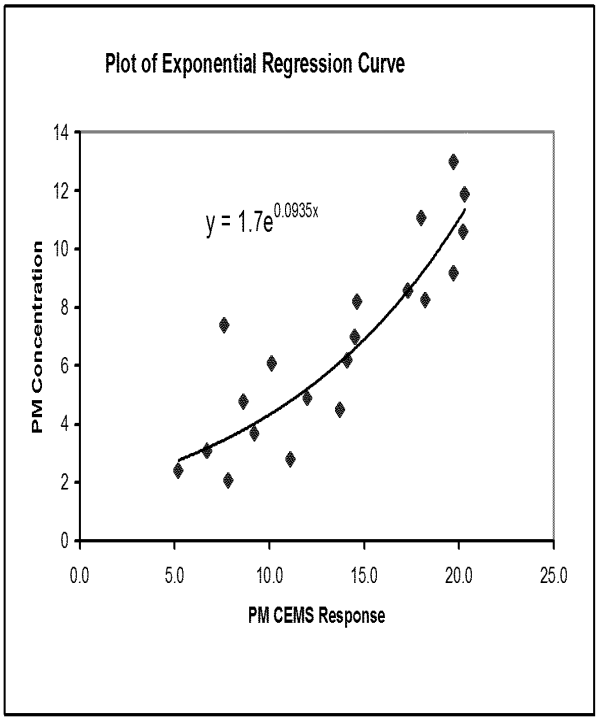
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 1.700\ e^{0.0935\ x}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.840	≥ 0.85	no
Confidence interval	4.52%	≤ 10%	yes
Tolerance interval	16.3%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



PS-11 Correlation Test
Calculation Spreadsheet
Version 2-6 10/25/04

CALCULATIONS FOR POWER CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

	PM CEMS response		PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	Measured	Transformed					
Run	x	x' = Ln(x)	y	y' = Ln(y)	$(x'_i-x'^{\sim})^2$	$(y'_i-y'^{\sim})^2$	$(x'_i-x'^{\sim})(y'_i-y'^{\sim})$	y'^	$(y'_i{}^{\wedge}-y'_i)^2$
1	6.7	1.902	3.1	1.131	0.385	0.429	0.406	1.100	0.0010
2	7.6	2.028	7.4	2.001	0.244	0.046	-0.106	1.240	0.5802
3	7.8	2.054	2.1	0.742	0.219	1.091	0.489	1.269	0.2773
4	9.2	2.219	3.7	1.308	0.092	0.229	0.145	1.451	0.0204
5	10.1	2.313	6.1	1.808	0.044	0.000	-0.005	1.554	0.0645
6	11.1	2.407	2.8	1.030	0.013	0.573	0.087	1.659	0.3959
7	12	2.485	4.9	1.589	0.001	0.039	0.007	1.745	0.0243
8	18.2	2.901	8.3	2.116	0.144	0.109	0.125	2.206	0.0080
9	13.7	2.617	4.5	1.504	0.009	0.080	-0.027	1.892	0.1502
10	14.6	2.681	8.2	2.104	0.025	0.101	0.050	1.962	0.0202
11	19.7	2.981	13	2.565	0.210	0.606	0.357	2.293	0.0737
12	18	2.890	11.1	2.407	0.135	0.385	0.228	2.194	0.0455
13	20.3	3.011	11.9	2.477	0.238	0.476	0.337	2.327	0.0225
14	20.2	3.006	10.6	2.361	0.234	0.330	0.278	2.321	0.0016
15	19.7	2.981	9.2	2.219	0.210	0.187	0.198	2.293	0.0055
16	5.2	1.649	2.4	0.875	0.763	0.830	0.796	0.820	0.0031
17	8.6	2.152	4.8	1.569	0.137	0.047	0.081	1.377	0.0369
18	14.1	2.646	6.2	1.825	0.015	0.001	0.005	1.923	0.0098
19	14.5	2.674	7	1.946	0.023	0.025	0.024	1.954	0.0001
20	17.3	2.851	8.6	2.152	0.108	0.133	0.120	2.150	0.0000

RESULTS OF POWER CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	20
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.522
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	3.25
$y'^{\sim}$	y'~ = 1/n*(Sum of (y'_i)) =	1.786
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	5.72
$S_{x'y'}$	$S_{x'y'}$ = Sum((x'_i-x'~)(y'_i-y'~)) =	3.60
b_0'	b_0' = y'~-b_1x~ =	-1.004
b_0	b_0 = e^b_0' =	0.367
b_1	b_1 = $S_{x'y'}$/$S_{x'x'}$ =	1.106
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.311
$y'^{\wedge}_{mean}$	y'^ at mean x value =	1.786
t_f	t_f from table =	2.101
CI'	CI = t_f*SL*SQRT(1/n) =	0.146
LCL'	LCL' = y~' - CI'	1.640
UCL'	UCL' = y~' + CI'	1.933
CI	CI = (e^UCL' - e^LCL')/2	0.875
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	4.73%
n'	n' = n =	20
v_f	v_f from table =	1.384
$u_{n'}$	$u_{n'}$ from table =	1.177
k_T	k_T = $u_{n'}$*v_f =	1.629
TI'	TI' = $k_T S_L$ =	0.507
LTL'	LTL' = y~' - TI'	1.280
UTL'	UTL' = y~' + TI'	2.293
TI	TI = (e^UTL' - e^LTL')/2	3.154
TI%	TI% = TI/EL*100% =	17.0%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.549
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.679
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.824

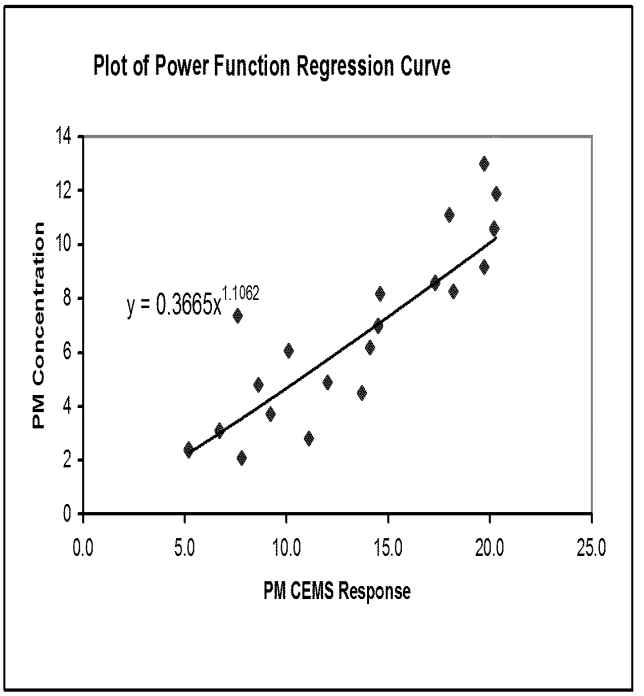
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 0.3665\ x^{1.1062}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.824	$\geq$ 0.85	no
Confidence interval	4.73%	$\leq$ 10%	yes
Tolerance interval	17.0%	$\leq$ 25%	yes

* Indicates correlation coefficient is undefined.



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SELECTION OF BEST MODEL

Facility: **Facility D** **Emission Unit:** **Boiler**
Location: **Anytown USA** **Test Dates:** **11/1-11/4/02, 1/7-1/8/03**

Model	Correlation coefficient (a)	≥ 0.85 ?	Confidence interval half range percentage	≤ 10% ?	Tolerance interval half range percentage	≤ 25% ?	Min/max within allowable range?	Does model meet all criteria?
Linear	0.861	Yes	4.25%	Yes	14.7%	Yes	(b)	Yes
Polynomial	0.875	Yes	5.47%	Yes	14.4%	Yes	Yes	Yes
Logarithmic	0.816	No	4.82%	Yes	16.7%	Yes	(b)	No
Exponential	0.840	No	4.52%	Yes	16.3%	Yes	(b)	No
Power	0.824	No	4.73%	Yes	17.0%	Yes	(b)	No

(a) * indicates correlation coefficient is undefined; model does not satisfy criterion.
(b) Not applicable; criterion applies only to polynomial model.

Best model: **Polynomial**

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Appendix A.6

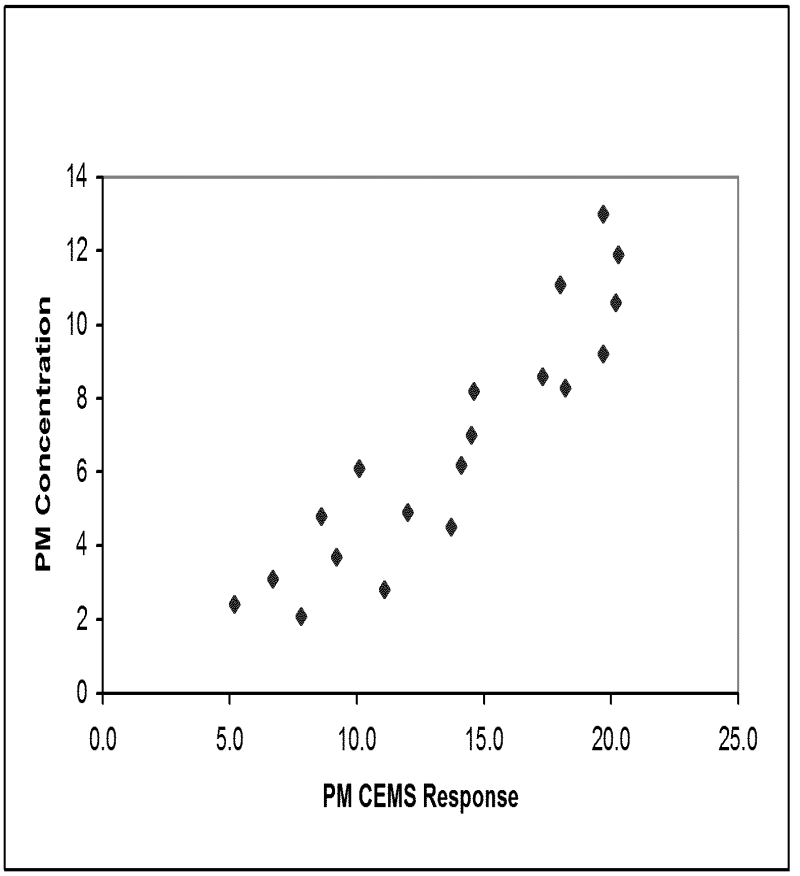
Complete Results of Example No. 4 – With Run 2 Excluded

CORRELATION TEST PM CEMS AND REFERENCE METHOD TEST DATA

Run	PM CEMS response x	PM concentration mg/acm y
1	6.7	3.1
3	7.8	2.1
4	9.2	3.7
5	10.1	6.1
6	11.1	2.8
7	12.0	4.9
8	18.2	8.3
9	13.7	4.5
10	14.6	8.2
11	19.7	13
12	18.0	11.1
13	20.3	11.9
14	20.2	10.6
15	19.7	9.2
16	5.2	2.4
17	8.6	4.8
18	14.1	6.2
19	14.5	7
20	17.3	8.6

Facility: Facility D
Location: Anytown USA
Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

	Value	Units
Emission limit:	18.5	mg/acm



CALCULATIONS FOR LINEAR CORRELATION

Facility: Facility D
Location: Anytown USA
Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Run	PM CEMS response x	PM concentration mg/acm y	CALCULATED STATISTICAL PARAMETERS				
			$(x_i-\tilde{x})^2$	$(y_i-\tilde{y})^2$	$(x_i-\tilde{x})(y_i-\tilde{y})$	$\hat{y}$	$(\hat{y}_i-y_i)^2$
1	6.7	3.1	49.52	13.42	25.78	2.338	0.581
3	7.8	2.1	35.25	21.75	27.68	3.029	0.864
4	9.2	3.7	20.58	9.38	13.90	3.910	0.044
5	10.1	6.1	13.23	0.44	2.41	4.476	2.638
6	11.1	2.8	6.95	15.71	10.45	5.105	5.312
7	12	4.9	3.02	3.47	3.24	5.671	0.594
8	18.2	8.3	19.92	2.36	6.86	9.570	1.613
9	13.7	4.5	0.00	5.12	0.08	6.740	5.018
10	14.6	8.2	0.75	2.06	1.24	7.306	0.799
11	19.7	13	35.56	38.90	37.19	10.513	6.183
12	18	11.1	18.17	18.81	18.49	9.444	2.741
13	20.3	11.9	43.08	26.39	33.71	10.891	1.019
14	20.2	10.6	41.77	14.72	24.80	10.828	0.052
15	19.7	9.2	35.56	5.94	14.53	10.513	1.725
16	5.2	2.4	72.88	19.04	37.25	1.394	1.011
17	8.6	4.8	26.39	3.85	10.08	3.533	1.606
18	14.1	6.2	0.13	0.32	-0.20	6.992	0.627
19	14.5	7	0.58	0.06	0.18	7.243	0.059
20	17.3	8.6	12.70	3.37	6.54	9.004	0.163

RESULTS OF LINEAR CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	19
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	13.74
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	436
$\tilde{y}$	y~ = 1/n*(Sum of (y_i)) =	6.76
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	205
S_{xy}	S_{xy} = Sum((x_i-x~)(y_i-y~)) =	274.2
b_0	b_0 = y~-b_1x~ =	-1.876
b_1	b_1 = S_{xy}/S_{xx} =	0.629
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	1.386
$y^{\wedge}_{mean}$	y^ at mean x value =	6.763
t_f	t_f from table =	2.110
CI	CI = t_f*S_L*SQRT(1/n) =	0.671
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	3.63%
n'	n' = n =	19
v_f	v_f from table =	1.400
$u_{n'}$	$u_{n'}$ from table =	1.178
k_T	k_t = u_n'*v_f =	1.649
TI	TI = k_t*S_L =	2.285
TI%	TI% = TI/EL*100% =	12.4%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	3.376
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.831
r	r = SQRT((1-S_L^2/S_y^2)) =	0.912

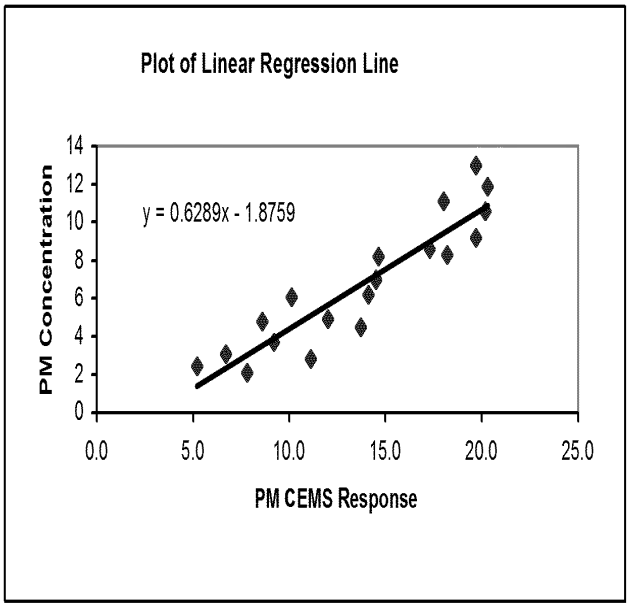
* Indicates correlation coefficient is undefined.

Correlation equation: y = -1.876 + 0.629 x

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.912	≥ 0.85	yes
Confidence interval	3.63%	≤ 10%	yes
Tolerance interval	12.4%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



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CALCULATIONS FOR POLYNOMIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

	PM CEMS response	PM concentration mg/acm	CALCULATED STATISTICAL PARAMETERS									
Run	x	y	x^2	x^3	x^4	xy	x^2y	y^	$(y^\wedge-y)^2$	delta	$(yi-y\sim)^2$	CI
1	6.7	3.1	45	301	2,015	21	139	2.906	0.038	0.2329	13.42	1.35
3	7.8	2.1	61	475	3,702	16	128	3.246	1.313	0.1432	21.75	1.06
4	9.2	3.7	85	779	7,164	34	313	3.770	0.005	0.1039	9.38	0.90
5	10.1	6.1	102	1,030	10,406	62	622	4.161	3.761	0.1035	0.44	0.90
6	11.1	2.8	123	1,368	15,181	31	345	4.644	3.400	0.1125	15.71	0.94
7	12	4.9	144	1,728	20,736	59	706	5.123	0.050	0.1215	3.47	0.97
8	18.2	8.3	331	6,029	109,720	151	2,749	9.569	1.610	0.0983	2.36	0.88
9	13.7	4.5	188	2,571	35,228	62	845	6.144	2.702	0.1262	5.12	0.99
10	14.6	8.2	213	3,112	45,437	120	1,748	6.745	2.118	0.1195	2.06	0.97
11	19.7	13	388	7,645	150,614	256	5,045	10.944	4.227	0.1725	38.90	1.16
12	18	11.1	324	5,832	104,976	200	3,596	9.394	2.910	0.0948	18.81	0.86
13	20.3	11.9	412	8,365	169,818	242	4,904	11.527	0.139	0.2351	26.39	1.36
14	20.2	10.6	408	8,242	166,497	214	4,325	11.428	0.686	0.2230	14.72	1.32
15	19.7	9.2	388	7,645	150,614	181	3,570	10.944	3.042	0.1725	5.94	1.16
16	5.2	2.4	27	141	731	12	65	2.543	0.020	0.4924	19.04	1.96
17	8.6	4.8	74	636	5,470	41	355	3.533	1.606	0.1131	3.85	0.94
18	14.1	6.2	199	2,803	39,525	87	1,233	6.406	0.042	0.1239	0.32	0.98
19	14.5	7	210	3,049	44,205	102	1,472	6.676	0.105	0.1205	0.06	0.97
20	17.3	8.6	299	5,178	89,575	149	2,574	8.800	0.040	0.0904	3.37	0.84

RESULTS OF POLYNOMIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	19
S_1	S1 = Sum (xi) =	261
S_2	S2 = Sum (xi2) =	4,021
S_3	S3 = Sum (xi3) =	66,929
S_4	S4 = Sum (xi4) =	1,171,613
S_5	S5 = Sum (yi) =	129
S_6	S6 = Sum (xiyi) =	2,039
S_7	S7 = Sum (xi2yi) =	34,734
det A	det A = nS2S4-S2S2S2+S1S3S2-S3S3n+S2S1S3-S4S1S1 =	5.945E+07
b_0	b0 = (S5S2S4+S1S3S7+S2S6S3-S7S2S2-S3S3S5-S4S6S1)/det A =	2.189
b_1	b1 = (nS6S4+S5S3S2+S2S1S7-S2S6S2-S7S3n-S4S1S5)/det A =	-0.067
b_2	b2 = (nS2S7+S1S6S2+S5S1S3-S2S2S5-S3S6n-S7S1S1)/det A =	0.02596
S_p	S_p = Sqrt((1/(n-3)Sum of (y^-Y)2) =	1.318
D	D = n(S2S4-S3^2)+S1(S3S2-S1S4)+S2(S1S3-S2^2) =	5.945E+07
C_0	C0 = (S2S4-S3^2)/D =	3.902
C_1	C1 = (S3S2-S1S4)/D =	-0.6164
C_2	C2 = (S1S3-S2^2)/D =	2.182E-02
C_3	C3 = (nS4-S2^2)/D =	1.024E-01
C_4	C4 = (S1S2-nS3)/D =	-3.736E-03
C_5	C5 = (nS2-S1^2)/D =	1.394E-04
t_f	t_f from table =	2.120
EL	Emission Limit =	18.5
CI	CI = t_f*S_p *sqrt(delta-min) =	0.840
CI%	CI% = CI/EL*100=	4.54%
v_f	v_f from table =	1.418
$u_{n'}$	$u_{n'}$ from table =	1.195
n'	n' = 1/delta =	11.06
k_T	kt = un'*vf =	1.694
TI	TI = kt*sp =	2.233
TI%	TI% = TI/EL *100% =	12.1%
y~	y~ = 1/n*(Sum of (yi)) =	6.76
S_y	Sy = sqrt(Sum of (yi-y~)^2/(n-1) =	3.38
r^2	r^2 = 1-(S_p^2/Sy^2) =	0.847
r	r = sqrt((1-S_p^2/Sy^2)) =	0.921
Max-min?	b_2 > 0?	minimum
$x_{max-min}$	y = -b_1/2b_2 =	1.29
1.25x_{max}		25.38

* Indicates correlation coefficient is undefined.

Correlation equation: $y = 2.189 + -0.067\ x + 0.02596\ x^2$

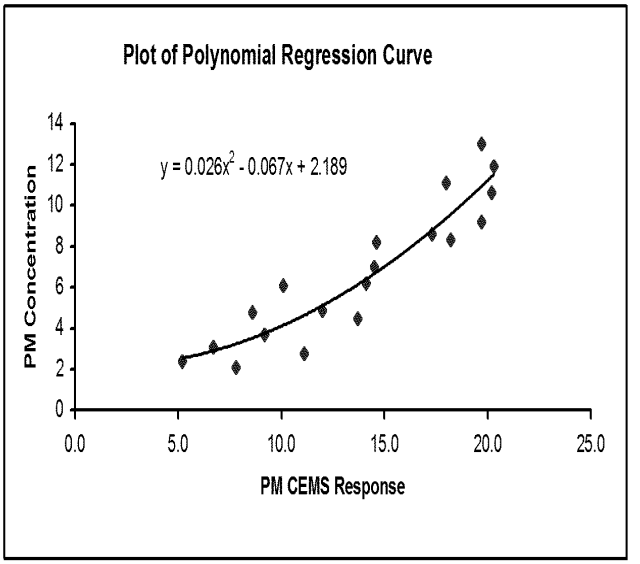
Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.921	≥ 0.85	yes
Confidence interval	4.54%	≤ 10%	yes
Tolerance interval	12.1%	≤ 25%	yes

* Indicates correlation coefficient is undefined.

Check for Correlation Curve Minimum/Maximum

Correlation curve minimum point	1.29
Minimum allowable x value	5.2
Is correlation curve minimum < minimum x value?	yes
Correlation curve maximum point	NA
Extrapolation limit for x (125% of maximum x value)	NA
Is correlation curve maximum > extrapolation limit?	NA



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CALCULATIONS FOR LOGARITHMIC CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

	PM CEMS response		PM concentration	CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	mg/acm					
Run	x	x' = ln (x)	y	$(x_i-\bar{x'})^2$	$(y_i-\bar{y})^2$	$(x'_i-\bar{x'})(y_i-\bar{y})$	y^	$(y\hat{}_i-y_i)^2$
1	6.7	1.902	3.1	0.418	13.419	2.367	2.082	1.037
3	7.8	2.054	2.1	0.244	21.745	2.305	3.183	1.173
4	9.2	2.219	3.7	0.108	9.383	1.008	4.379	0.461
5	10.1	2.313	6.1	0.056	0.440	0.156	5.055	1.092
6	11.1	2.407	2.8	0.020	15.707	0.560	5.739	8.636
7	12	2.485	4.9	0.004	3.471	0.118	6.303	1.970
8	18.2	2.901	8.3	0.125	2.362	0.543	9.321	1.042
9	13.7	2.617	4.5	0.005	5.122	-0.156	7.263	7.635
10	14.6	2.681	8.2	0.018	2.065	0.191	7.724	0.226
11	19.7	2.981	13	0.187	38.898	2.696	9.894	9.645
12	18	2.890	11.1	0.117	18.808	1.483	9.241	3.457
13	20.3	3.011	11.9	0.214	26.387	2.375	10.112	3.198
14	20.2	3.006	10.6	0.209	14.721	1.755	10.076	0.275
15	19.7	2.981	9.2	0.187	5.938	1.053	9.894	0.482
16	5.2	1.649	2.4	0.809	19.037	3.926	0.246	4.641
17	8.6	2.152	4.8	0.157	3.854	0.779	3.890	0.828
18	14.1	2.646	6.2	0.010	0.317	-0.055	7.472	1.617
19	14.5	2.674	7	0.016	0.056	0.030	7.674	0.455
20	17.3	2.851	8.6	0.091	3.374	0.555	8.953	0.125

RESULTS OF LOGARITHMIC CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	19
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.55
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	2.99
$y^{\sim}$	y~ = 1/n*(Sum of (y_i)) =	6.8
S_{yy}	S_{yy} = Sum((y_i-y~)^2) =	205
$S_{x'y}$	$S_{x'y}$ = Sum((x'_i-x'~)(y_i-y~)) =	21.7
b_0	b_0 = y~-b_1x'~ =	-11.697
b_1	b_1 = $S_{x'y}$/$S_{x'x'}$ =	7.244
S_L	S_L = SQRT(1/(n-2)(Sum(y_i^-y_i)^2)) =	1.680
$y^{\wedge}_{mean}$	y^ at mean x value =	6.763
t_f	t_f from table =	2.110
CI	CI = t_f*S_L*SQRT(1/n) =	0.813
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	4.40%
n'	n' = n =	19
v_f	v_f from table =	1.400
$u_{n'}$	$u_{n'}$ from table =	1.178
k_T	k_t = u_n'*v_f =	1.649
TI	TI = k_t*SL =	2.77
TI%	TI% = TI/EL*100% =	15.0%
S_y	S_y = SQRT(S_{yy}/(n-1)) =	3.376
r^2	r^2 = 1-(S_L^2/S_y^2) =	0.752
r	r = SQRT((1-S_L^2/S_y^2)) =	0.867

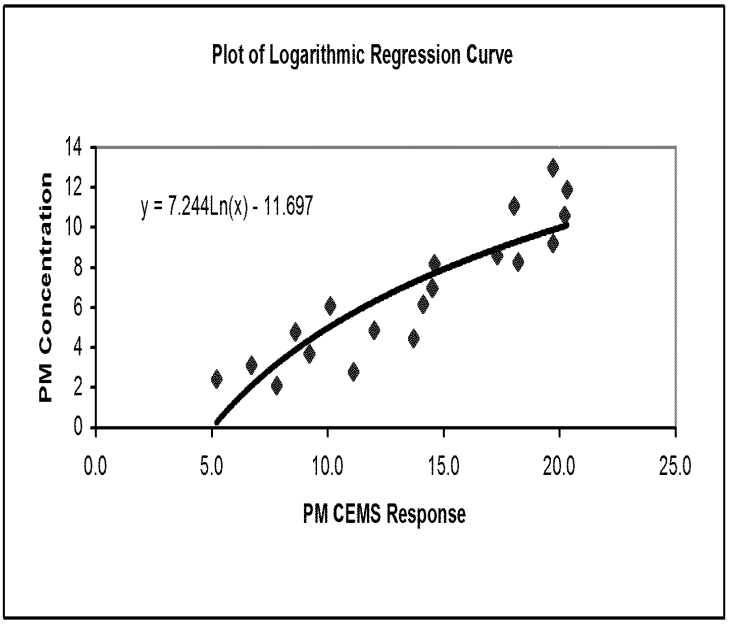
* Indicates correlation coefficient is undefined.

Correlation equation: y = -11.697 + 7.244 Ln(x)

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.867	≥ 0.85	yes
Confidence interval	4.40%	≤ 10%	yes
Tolerance interval	15.0%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



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CALCULATIONS FOR EXPONENTIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Run	PM CEMS response x	PM concentration mg/acm Measured y	PM concentration mg/acm Transformed y' = Ln(y)	CALCULATED STATISTICAL PARAMETERS $(x_i-\bar{x})^2$	$(y'_i-\bar{y'})^2$	$(x_i-\bar{x})(y'_i-\bar{y'})$	$y'^{\wedge}$	$(y'^{\wedge}_i-y'_i)^2$
1	6.7	3.1	1.131	49.52	0.414	4.530	1.042	0.008
3	7.8	2.1	0.742	35.25	1.068	6.134	1.156	0.172
4	9.2	3.7	1.308	20.58	0.218	2.118	1.302	0.000
5	10.1	6.1	1.808	13.23	0.001	-0.120	1.396	0.170
6	11.1	2.8	1.030	6.95	0.556	1.966	1.500	0.222
7	12	4.9	1.589	3.02	0.035	0.323	1.594	0.000
8	18.2	8.3	2.116	19.92	0.116	1.522	2.240	0.015
9	13.7	4.5	1.504	0.00	0.073	0.010	1.771	0.071
10	14.6	8.2	2.104	0.75	0.108	0.284	1.865	0.057
11	19.7	13	2.565	35.56	0.624	4.710	2.397	0.028
12	18	11.1	2.407	18.17	0.399	2.693	2.219	0.035
13	20.3	11.9	2.477	43.08	0.492	4.603	2.459	0.000
14	20.2	10.6	2.361	41.77	0.343	3.785	2.449	0.008
15	19.7	9.2	2.219	35.56	0.197	2.648	2.397	0.031
16	5.2	2.4	0.875	72.88	0.809	7.681	0.886	0.000
17	8.6	4.8	1.569	26.39	0.043	1.061	1.240	0.108
18	14.1	6.2	1.825	0.13	0.002	0.018	1.813	0.000
19	14.5	7	1.946	0.58	0.029	0.130	1.855	0.008
20	17.3	8.6	2.152	12.70	0.142	1.342	2.146	0.000

RESULTS OF EXPONENTIAL CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	19
$\tilde{x}$	x~ = 1/n*(Sum of (x_i)) =	13.74
S_{xx}	S_{xx} = Sum((x_i-x~)^2) =	436.0
$y'^{\sim}$	y'~ = 1/n*(Sum of (y'_i)) =	1.775
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	5.67
$S_{xy'}$	$S_{xy'}$ = Sum((x_i-x~)(y'_i-y'~)) =	45.44
b_0'	b_0' = y'~-b_1x~ =	0.344
b_0	b_0 = e^b_0' =	1.410
b_1	b_1 = $S_{xy'}$/S_{xx} =	0.104
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.235
$y'^{\wedge}_{mean}$	y'^ at mean x value =	1.775
t_f	t_f from table =	2.110
CI'	CI' = t_f*SL*SQRT(1/n) =	0.1135
LCL'	LCL' = y~' - CI'	1.662
UCL'	UCL' = y~' + CI'	1.889
CI	CI = (e^UCL' - e^LCL')/2	0.671
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	3.63%
n'	n' = n =	19
v_f	v_f from table =	1.400
$u_{n'}$	$u_{n'}$ from table =	1.178
k_T	k_T = u_n'*v_f =	1.649
TI'	TI' = k_t*SL =	0.387
LTL'	LTL' = y~' - TI'	1.388
UTL'	UTL' = y~' + TI'	2.162
TI	TI = (e^UTL' - e^LTL')/2	2.340
TI%	TI% = TI/EL*100% =	12.6%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.561
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.825
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.909

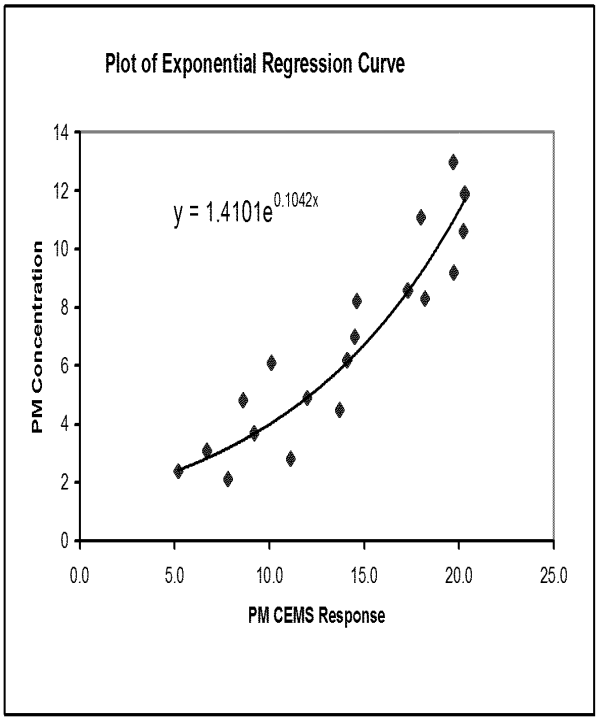
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 1.410\ e^{0.1042\ x}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.909	≥ 0.85	yes
Confidence interval	3.63%	≤ 10%	yes
Tolerance interval	12.6%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



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CALCULATIONS FOR POWER CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

	PM CEMS response		PM concentration mg/acm		CALCULATED STATISTICAL PARAMETERS				
	Measured	Transformed	Measured	Transformed					
Run	x	x' = Ln(x)	y	y' = Ln(y)	$(x'_i-x'^{\sim})^2$	$(y'_i-y'^{\sim})^2$	$(x'_i-x'^{\sim})(y'_i-y'^{\sim})$	y'^	$(y'_i{}^{\wedge}-y'_i)^2$
1	6.7	1.902	3.1	1.131	0.418	0.414	0.416	0.975	0.0245
3	7.8	2.054	2.1	0.742	0.244	1.068	0.511	1.163	0.1773
4	9.2	2.219	3.7	1.308	0.108	0.218	0.154	1.367	0.0035
5	10.1	2.313	6.1	1.808	0.056	0.001	-0.008	1.483	0.1058
6	11.1	2.407	2.8	1.030	0.020	0.556	0.105	1.600	0.3253
7	12	2.485	4.9	1.589	0.004	0.035	0.012	1.697	0.0115
8	18.2	2.901	8.3	2.116	0.125	0.116	0.120	2.212	0.0092
9	13.7	2.617	4.5	1.504	0.005	0.073	-0.019	1.861	0.1271
10	14.6	2.681	8.2	2.104	0.018	0.108	0.044	1.939	0.0271
11	19.7	2.981	13	2.565	0.187	0.624	0.341	2.311	0.0647
12	18	2.890	11.1	2.407	0.117	0.399	0.216	2.199	0.0433
13	20.3	3.011	11.9	2.477	0.214	0.492	0.324	2.348	0.0166
14	20.2	3.006	10.6	2.361	0.209	0.343	0.268	2.342	0.0004
15	19.7	2.981	9.2	2.219	0.187	0.197	0.192	2.311	0.0083
16	5.2	1.649	2.4	0.875	0.809	0.809	0.809	0.661	0.0461
17	8.6	2.152	4.8	1.569	0.157	0.043	0.082	1.284	0.0810
18	14.1	2.646	6.2	1.825	0.010	0.002	0.005	1.896	0.0051
19	14.5	2.674	7	1.946	0.016	0.029	0.021	1.931	0.0002
20	17.3	2.851	8.6	2.152	0.091	0.142	0.114	2.150	0.0000

RESULTS OF POWER CORRELATION

Facility: Facility D
Location: Anytown USA

Emission Unit: Boiler
Test Dates: 11/1-11/4/02, 1/7-1/8/03

Variable	Equation	Value
n	Number of data points =	19
$x'^{\sim}$	x'~ = 1/n*(Sum of (x'_i)) =	2.548
$S_{x'x'}$	$S_{x'x'}$ = Sum((x'_i-x'~)^2) =	2.99
$y'^{\sim}$	y'~ = 1/n*(Sum of (y'_i)) =	1.775
$S_{y'y'}$	$S_{y'y'}$ = Sum((y'_i-y'~)^2) =	5.67
$S_{x'y'}$	$S_{x'y'}$ = Sum((x'_i-x'~)(y'_i-y'~)) =	3.71
b_0'	b_0' = y'~-b_1x~ =	-1.381
b_0	b_0 = e^b_0' =	0.251
b_1	b_1 = $S_{x'y'}$/$S_{x'x'}$ =	1.239
S_L	S_L = SQRT(1/(n-2)(Sum(y'_i^-y'_i)^2)) =	0.252
$y'^{\wedge}_{mean}$	y'^ at mean x value =	1.775
t_f	t_f from table =	2.110
CI'	CI = t_f*SL*SQRT(1/n) =	0.122
LCL'	LCL' = y~' - CI'	1.653
UCL'	UCL' = y~' + CI'	1.897
CI	CI = (e^UCL' - e^LCL')/2	0.721
EL	Emission Limit =	18.5
CI%	CI% =CI/EL*100% =	3.90%
n'	n' = n =	19
v_f	v_f from table =	1.400
$u_{n'}$	$u_{n'}$ from table =	1.178
k_T	k_T = $u_{n'}$*v_f =	1.649
TI'	TI' = $k_T S_L$ =	0.415
LTL'	LTL' = y~' - TI'	1.360
UTL'	UTL' = y~' + TI'	2.190
TI	TI = (e^UTL' - e^LTL')/2	2.521
TI%	TI% = TI/EL*100% =	13.6%
$S_{y'}$	$S_{y'}$ = SQRT($S_{y'y'}$/(n-1)) =	0.561
r^2	r^2 = 1-(SL^2/$S_{y'}$^2) =	0.799
r	r = SQRT((1-SL^2/$S_{y'}$^2)) =	0.894

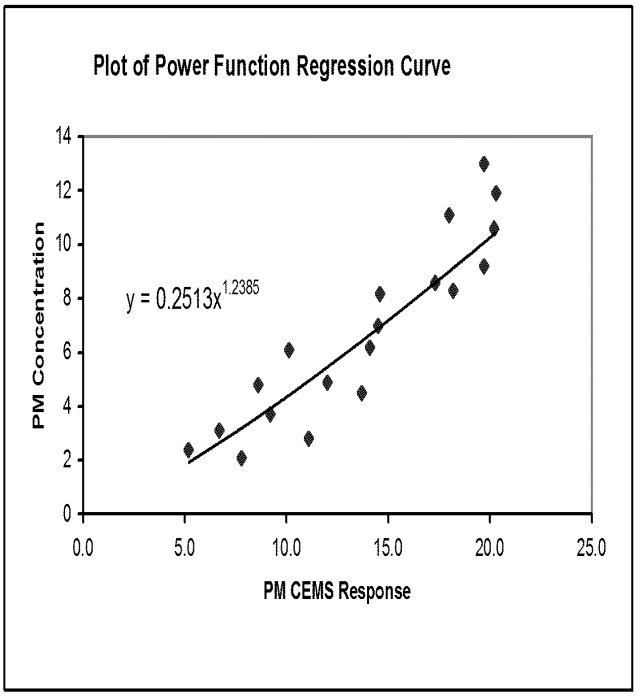
* Indicates correlation coefficient is undefined.

Correlation equation: $y = 0.2513\ x^{1.2385}$

Summary of Acceptance Criteria for PS-11

Criterion	Actual	Allowable	Acceptable?
Correlation coefficient	0.894	≥ 0.85	yes
Confidence interval	3.90%	≤ 10%	yes
Tolerance interval	13.6%	≤ 25%	yes

* Indicates correlation coefficient is undefined.



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SELECTION OF BEST MODEL

Facility: **Facility D** **Emission Unit:** **Boiler**
Location: **Anytown USA** **Test Dates:** **11/1-11/4/02, 1/7-1/8/03**

Model	Correlation coefficient (a)	≥ 0.85 ?	Confidence interval half range percentage	≤ 10% ?	Tolerance interval half range percentage	≤ 25% ?	Min/max within allowable range?	Does model meet all criteria?
Linear	0.912	Yes	3.63%	Yes	12.4%	Yes	(b)	Yes
Polynomial	0.921	Yes	4.54%	Yes	12.1%	Yes	Yes	Yes
Logarithmic	0.867	Yes	4.40%	Yes	15.0%	Yes	(b)	Yes
Exponential	0.909	Yes	3.63%	Yes	12.6%	Yes	(b)	Yes
Power	0.894	Yes	3.90%	Yes	13.6%	Yes	(b)	Yes

(a) * indicates correlation coefficient is undefined; model does not satisfy criterion.
(b) Not applicable; criterion applies only to polynomial model.

Best model: **Polynomial**

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