1d plume model based on equations in Joe Kordzi's Basic program. The solution is not coupled, but instead individual components are added together. Individual parts include regional groundwater velocity, operational plume, buoyancy, dispersion, and diffusion.

Facility: Corsicana Technologies
Case: 1 - k=191 md

1. Define Units

$cp := .01 \cdot poise$

$g = 9.807\, m \cdot s^{-2}$

$md := 7.32441 \cdot 10^{-8} \cdot \dfrac{\dfrac{ft^3 \cdot cp}{sec}}{ft^2 \cdot \dfrac{psi}{ft}}$

$gal := 0.1336894\, ft^3$

$acre := 43560 \cdot ft^2$

2. Reservoir and 10,000 Year Plume Demonstration Parameters

$\rho go := 61.97 \cdot \dfrac{lb}{ft^3}$

$\rho gi := 61.61 \cdot \dfrac{lb}{ft^3}$

$k := 191 \cdot md$ permeability

$\rho gi = 0.987 \dfrac{gm}{cm^3}$ injectate density

$\phi := 0.20$ porosity

$\rho go = 0.993 \dfrac{gm}{cm^3}$ formation fluid density

$h := 36 \cdot ft$ net thickness

$cumvolume := 50 \cdot \dfrac{gal}{min} \cdot 30 \cdot yr$

$dip := 200 \cdot \dfrac{ft}{2.355 \cdot mi}$

$\mu := 0.479 \cdot cp$ viscosity

$cumvolume = 7.889 \times 10^8\, gal$ cumulative injection volume

$\Delta t := 500 \cdot yr$ plume drift time

$\theta := 0.92146586\, deg$

$\sin(\theta) = 0.016082$

$\alpha T := 16 \cdot ft$ transverse dispersivity

$D0 := 4.8 \cdot 10^{-5} \cdot \dfrac{cm^2}{s}$ free water diffusivity

$\alpha L := 160 \cdot ft$ longitudinal dispersity

$CRF := 1$ concentration reduction factor

$Vdrift := 0 \cdot \dfrac{ft}{yr}$ regional groundwater velocity

$\tau := 1$ tortuosity

3. Operational Plume Radius and Area

$$operational_plume_radius := \sqrt{\frac{cumvolume}{\pi \cdot \phi \cdot h}}$$

$operational_plume_radius = 2.123 \times 10^3\, ft$ calculated operational plume radius

$Area_of_plume := \pi \cdot operational_plume_radius^2$

$Area_of_plume = 14153336.4 ft^2$ calculated operational plume area

$Area_of_plume = 324.916 acre$

4. Movement due to Regional Ground Water Velocity

$\Delta t = 500 yr$ $Vdrift := 0 \cdot \frac{ft}{yr}$

$Ground_water_movement_distance = Vdrift \cdot \Delta t$

$Ground_water_movement_distance = 0 ft$ calculated movement from regional velocity

5. Movement due to density drift from bouyancy

$Den1 := 4 \cdot \pi \cdot \sqrt{\alpha T \cdot \alpha L} \cdot k \cdot |\rho gi - \rho go| \cdot g \cdot \sin(\theta) \cdot \Delta t$

$Den1 = 1.72\, kg \cdot s^{-1} ft$

$Den2 := \phi^2 \cdot \mu \cdot Area_of_plume$

$Den2 = 88.542\, kg \cdot s^{-1} ft$

$Den3 := 4 \cdot \pi \cdot \frac{\sqrt{\alpha T \cdot \alpha L}}{Area_of_plume \cdot \phi}$

$Den3 = 2.17 \times 10^{-4} ft^{-1}$

$$Density_drift_distance := \frac{\left[\left[1 + \left(\frac{Den1}{Den2}\right)\right]^{0.5}\right] - 1}{Den3}$$

$Density_drift_distance = 44.5 ft$ calculated plume movement from bouyant drift

6. Movement due to dispersion and diffusion

$\tau := 1$ tortuosity $D0 = 4.8 \times 10^{-9}\, m^2 \cdot s^{-1}$ free water diffusivity

$CRF = 1$ concentration reduction factor

$L := Density_drift_distance + operational_plume_radius$

$L = 2167.07ft$ operational plume radius and buoyant plume length

$standard_deviation := [2 \cdot [(\alpha L \cdot L) + (D0 \cdot \tau \cdot \Delta t)]]^{0.5}$

$standard_deviation = 833.7ft$

Iterative Calculations for error function

$i := 0 .. 6000$ $X_i := 0.01 + (i \cdot 0.001)$ $X_0 = 0.01$

$Error_CRF := 2 \cdot (CRF)$ $Error_CRF = 2$

$Error_function_i := erfc(X_i)$ $Error_function_0 = 0.98871658444415$

$Difference_i := Error_function_i - Error_CRF$ $Difference_0 = -1.011$

$$Converge(X, tol) := \left| \begin{array}{l} j \leftarrow 0 \\ \text{while } Difference_j > tol \\ \quad j \leftarrow j + 1 \\ X_j \end{array} \right.$$

Programming loop to determine argument - solve inverse complementary error function

Define arrays for loop

$Converge(X, 0.000000000000) = 0.01$

$Z := Converge(X, 0.0000000000000)$

$Z = 0.01$ The argument of the inverse complementary error function

$\Delta r := \sqrt{2} \cdot Z \cdot standard_deviation$ from the following equation: $CRF := erfc\left(\frac{\Delta r}{standard_deviation \cdot \sqrt{2}}\right)$

$\Delta r = 11.8ft$ diffusive and dispersive plume component

7. Summary of Results

$operational_plume_radius = 2122.5ft$ calculated operational plume radius

$Ground_water_movement_distance = 0ft$ calculated movement from regional velocity

$Density_drift_distance = 44.5ft$ calculated plume movement from bouyant drift

$\Delta r = 11.8ft$ diffusive and dispersive plume movement

$total_plume_distance := operational_plume_radius + Ground_water_movement_distance + Density_drift_distance$

total_plume_distance = 2178.9ft

total plume movement from all effects