Interpolation of Relative Permeability and Capillary Pressure – Method 1 (Optional)

PURPOSE:

Specify method-1 for interpolation of saturation functions as directed by keyword **`*KRINTRP`**.

FORMAT:

```
*INTERP_SCAL (*ON | *OFF)
```

DEFINITIONS:

`*INTERP_SCAL`

Use a two stage method for interpolation between two sets of rock-fluid curves. First, the saturation (horizontal axis) endpoints are interpolated separately and applied to the two normalized curves. Then the curve values (vertical axes) are obtained at the required saturation. The curve values are obtained at the required saturation, for each set of curves separately, which are then interpolated to give the final result. If `*INTERP_SCAL` is entered in the model, **`*INTERP_ENDS *OFF`** is enforced.

DEFAULTS:

`*INTERP_SCAL *ON` is the default for interpolation of relative permeability and capillary pressure.

If `*INTERP_SCAL` appears without `*ON` or `*OFF`, `*ON` is assumed.

CONDITIONS:

The interpolation method specified by `*INTERP_SCAL` or its default applies to all rock types, that is all rock types use the same method.

EXPLANATION:

`*INTERP_SCAL` supersedes **`*INTERP_ENDS`** which was the only method available prior to release of GEM version 2018.10. `*INTERP_SCAL` utilizes a superior algorithm for end point interpolation and is the preferred method. It provides consistent interpolation results for relative permeability and capillary pressure curves even if their saturation endpoints are different.

Under certain circumstances, such as nearly miscible fluids, pH changes, surfactant concentration changes, large increases in applied flow velocities, polymer flooding and foamy oil, etc., the assumption that saturation function (relative permeability and capillary pressure) are functions only of fluid saturations and saturation history (hysteresis) is not sufficient to accurately describe observed flow behavior.

In these cases, the ability to interpolate basic relative permeability and capillary pressure data as functions of fluid properties, such as component concentration or flow capillary-number, can prove very useful. Because of the flexibility in the choice of interpolation related parameters and the fact that arbitrary tabular relative permeability and capillary pressure can be employed, a wide variety of physical effects mentioned above can be modeled.

In the following, the general concept of interpolation is discussed taking water relative permeability (k_{rw}) as an example. And the formulation for other saturation functions is given afterwards for reference.

In Figure 1 below, two k_{rw} curves labelled A and B exhibit different shape and endpoints (the critical water saturation S_{wcrit} and irreducible oil saturation S_{oirw}). Assuming that the

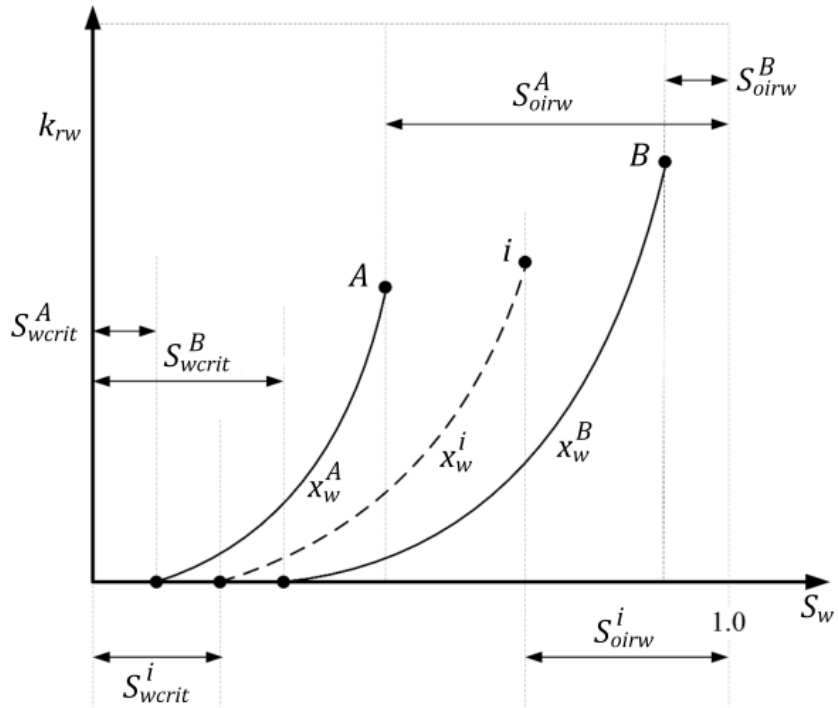

Figure 1: Sketch showing k_{rw} interpolation between curve A and B with respect to x

interpolation variable in this case is a component phase mole fraction x and the value of x associated with curve A and B for k_{rw} is x^A_w and x^B_w (called interpolation parameters and introduced by ***DTRAPW**), further assuming that a grid block has a mole fraction x^i and $x^i \in [x^A_w, x^B_w]$, the k_{rw} value of the block is obtained by the following procedure. First, an interpolation factor ω^i_w is computed by Equation (1)

$$\omega^i_w = \left(\frac{x^i - x^A_w}{x^B_w - x^A_w}\right)^{n_w} \tag{1}$$

n_w in Equation (1) is a curvature exponent (defined by ***WCRV**) and its effect on the final interpolation result will be discussed later on. The saturation endpoints of the block are then computed by Equations (2) and (3)

$$S^i_{wcrit} = \left(1 - \omega^i_w\right)\, S^A_{wcrit} + \omega^i_w\, S^B_{wsrit} \tag{2}$$

$$S^i_{oirw} = \left(1 - \omega^i_w\right)\, S^A_{oirw} + \omega^i_w\, S^B_{oirw} \tag{3}$$

The table looking-up saturation S^A_w and S^B_w for curve A and B will need a scaling, or linear transformation, of the block saturation S_w,

$$S_w^A = S_{wcrit}^A + \left(S_w - S_{wcrit}^i\right)\left(\frac{1 - S_{oirw}^A - S_{wcrit}^A}{1 - S_{oirw}^i - S_{wcrit}^i}\right) \tag{4}$$

$$S_w^B = S_{wcrit}^B + \left(S_w - S_{wcrit}^i\right)\left(\frac{1 - S_{oirw}^B - S_{wcrit}^B}{1 - S_{oirw}^i - S_{wcrit}^i}\right) \tag{5}$$

The relative permeability k_{rw}^A and k_{rw}^B can be then determined by a straight table look-up via curve A and B according to S_w^A and S_w^B.

Finally, the block's water relative permeability k_{rw} is computed from Equation (6)

$$k_{rw} = k_{rw}\left(S_w, x^i\right) = \left(1 - \omega_w^i\right) k_{rw}^A + \omega_w^i k_{rw}^B \tag{6}$$

In a similar fashion, the relative permeability of other phases (k_{ro} and k_{rg}), and capillary pressure values (P_{cow} and P_{cog}) of the grid block are calculated from

$$k_{ro} = \left(1 - \omega_o^i\right) k_{ro}^A + \omega_o^i k_{ro}^B \tag{7}$$

$$k_{rg} = \left(1 - \omega_g^i\right) k_{rg}^A + \omega_g^i k_{rg}^B \tag{8}$$

$$P_{cow} = \left(1 - \omega_o^i\right) P_{cow}^A + \omega_o^i P_{cow}^B \tag{9}$$

$$P_{cog} = \left(1 - \omega_g^i\right) P_{cog}^A + \omega_g^i P_{cog}^B \tag{10}$$

The interpolation factors in the above equations are computed from

$$\omega_o^i = \left(\frac{x^i - x_o^A}{x_o^B - x_o^A}\right)^{n_o}, \quad \omega_g^i = \left(\frac{x^i - x_o^A}{x_o^B - x_o^A}\right)^{n_g} \tag{11}$$

In Equation (11), x^i is again the block's current interpolation variable; x_o^A and x_o^B; are the oil phase interpolation parameters defined by ***DTRAPW** on curve A and B; n_o and n_g are the curvature exponents defined by ***OCRV** and ***GCRV** for oil and gas phase, respectively.

It is worth reiterating that the block saturation endpoints S_{wcrit}^i and S_{oirw}^i discussed thus far have been described as a result of weighted averaging of user-entered table values Equations (2) and (3). However, if endpoints are being altered by one of the endpoint facilities (e.g., ***TSOIRW** and/or ***BSOIRW**), such averaging for S_{wcrit}^i and S_{oirw}^i should be performed between a pair of endpoint values of the block and each value of the pair represents the block's saturation endpoint at a particular interpolation parameter.

Several observations can be made from the interpolation equations above.

1. k_r and P_c are nonlinear functions of x^i except for curvature exponents of 1.
2. ω_i lies between 0 and 1 because x^i generally lies between x^A and x^B.
3. x^i may lie beyond the set boundary. If $x^i < x^A$ (or $x^i > x^A$ when $x^A > x^B$), $\omega_i = 0$ is enforced; if $x^i > x^B$, (or $x^i < x^B$ when $x^A > x^B$), $\omega_i = 1$ is enforced.
4. k_r (and P_c) equals to the value on curve A at $x^i = x^A$ and to the value on curve B at $x^i = x^B$ regardless of curvature exponent n.
5. At any given x^i, k_r and P_c edges toward curve A as n increases and toward curve B as n decreases. This fact can be exploited to achieve certain weighting effect so that interpolation curves will congregate in desired area.

`*INTERP_SCAL` and **`*INTERP_ENDS`** are mutually exclusive methods for end-point interpolation. `*INTERP_SCAL` is the preferred method.

To use `*INTERP_SCAL`

`*INTERP_SCAL *ON` is on by default, therefore no data changes are required.

To use `*INTERP_ENDS` enter the following keywords

```
*INTERP_SCAL *OFF
```

`*INTERP_ENDS *ON`.

To turn off end point interpolation entirely, enter the following keyword

```
*INTERP_SCAL *OFF
```

Do not enter **`*INTERP_ENDS`**

In this case end point interpolation will not be done. Equations (1) to (5) will not apply and the end points entered directly for k_r and P_c for curve A and curve B will be used to calculate k_{rw}^A and k_{rw}^B for example, and be used in Equations (6) to (10). The interpolation of curves (not the end points) will still take place, since that is controlled by keyword **`*INTCOMP`**.