Thermal Option in GEM

This section describes the development of an energy equation to calculate temperature distribution in the reservoir for the compositional processes where reservoir temperature could change with time, for instance when the injected fluid is at a different temperature than the one prevailing in the reservoir. The equations to be solved are:

- Volume constraint equation: the volume of fluids must equal the pore volume.
- Component flow equations: material balance equations for oil, gas and water components.
- Energy balance equation including convection, conduction and heat losses.
- Phase equilibrium equations.

The volume constraint, component flow and phase equilibrium equations are described in Appendix A. Here we focus on the energy balance equation and related aspects.

Energy Balance Equation

To calculate the temperature distribution for the thermal cases, the following energy balance equation is added to the equation set.

$$\begin{aligned}
&\psi_T \equiv \Delta T_o^m H_o^m \left(\Delta p_o^{n+1} - \tilde{\rho}_o^m g \Delta D\right) + \Delta T_g^m H_g^m \left(\Delta p_o^{n+1} + \Delta P_{cog}^m - \tilde{\rho}_g^m g \Delta D\right) \\
&+\Delta T_w^m H_w^m \left(\Delta p_o^{n+1} - \Delta P_{cwo}^m - \tilde{\rho}_w^m g \Delta D\right) + \Delta \tau_c^m \Delta T^{n+1} + Q_{loss} + \sum_k H_k^{n+1} q_k^{n+1} \\
&-\frac{V}{\Delta t}\left[\phi^{n+1}\left(\sum_k \rho_k^{n+1} S_k^{n+1} U_k^{n+1}\right) - \phi^n \left(\sum_k \rho_k^n s_k^n U_k^n\right)\right] \\
&-\frac{V}{\Delta t}\left[\left(1-\phi_0\right) c_R \tilde{\rho}_R \left(T^{n+1} - T^n\right)\right] \;=\; 0, \;\; k = o, g, w
\end{aligned} \tag{E1.1}$$

where

c_R	=	heat capacity of rock
H_k	=	molar enthalpy of Phase k(k = o,g,w)
S_k	=	saturation of Phase k(k = o,g,w)
Q_{loss}	=	heat loss rate to the (over/underburden) surroundings
T	=	temperature
ϕ	=	Porosity (0 = initial; n+1 = current timestep; n = previous timestep)
U_k	=	H_k - p/p_k; molar internal energy of Phase k
$\tilde{\rho}_R$	=	Rock mass density

τ_c = total thermal conductivity of rock and fluids

Equation (E1.1) is an energy balance equation involving convection, conduction and heat losses to the surroundings.

Enthalpy Calculations

The enthalpy of the water phase is calculated from a look-up of the steam table. The oil and gas enthalpies are calculated from an EOS as follows.

The excess enthalpy for a fluid, which is the difference of the enthalpy at p and T and the ideal gas enthalpy at zero pressure and T, can be calculated from an EOS:

$$\Delta H^E = H - H^* = RT\,(Z-1) + \int_{\infty}^{v} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p \right] dv \tag{E1.2}$$

where

R = universal gas constant

v = molar volume

Z = compressibility factor

The above quantity is also referred to as enthalpy departure. Using Equation (E1.2), the following equation can be derived to calculate enthalpy departure for the SRK or PR EOS:

$$\Delta H^E = RT\,(Z-1) + \frac{T\,(\partial a/\partial T) - a}{b\,(\delta_2 - \delta_1)}\ \ln\ \left(\frac{v+\delta_2 b}{v+\delta_1 b}\right) \tag{E1.3}$$

where

δ_1 = 1 - $\sqrt{2}$; δ_2 = 1 + $\sqrt{2}$ for the PR EOS, and

δ_1 = 0; δ_2 = 1 for the SRK EOS

and a and b are EOS parameters.

The *API Technical Data Book - Petroleum Refining V. 2 (1983)* provides polynomials for estimating pure component enthalpies, H_i^* in the form:

$$H_i^* = \sum_{j=0}^{5} a_{ji} T^j \tag{E1.4}$$

where a_{ji} are the polynomial coefficients for Component i. Knowing ΔH^E and H^*, the enthalpy H and the internal energy can be calculated from:

$$U_k = H_k - pv_k = H_k - p / \rho_k \tag{E1.5}$$

Calculation of Enthalpies for Pseudo Components

Kesler and Lee (1976) provide the following correlation for calculating ideal gas heat capacities for heavy-fraction pseudo components from their specific gravity and normal boiling point:

$$\begin{aligned} c_{pi}^* &= -0.32646 + 0.02678\, K_{uop} \\ &- \left(1.3892 - 1.2122\, K_{uop} + 0.03803\, K_{uop}^2\right) \cdot 10^{-4} T - 1.5393 \cdot 10^{-7} T^2 \\ &- C_F \left[0.084773 - 0.080809\, \rho' - \left(2.1773 - 2.0826\, \rho'\right) \cdot 10^{-4}\, T\right] \\ &+ C_F \left[\left(0.78649 - 0.70423\, \rho'\right) \cdot 10^{-7}\, T^2\right] \end{aligned} \tag{E1.6}$$

where ρ' is the specific gravity, T_b is the normal boiling point in °R, K_{uop} is the Watson characterization factor given by:

$$K_{uop} = (T_b)^{\frac{1}{3}} / \rho' \tag{E1.7}$$

and

$$C_F = \left[\left(\frac{12.8}{K_{uop}} - 1\right) \times \left(\frac{10}{K_{uop}} - 1\right) \times 100\right]^2 \tag{E1.8}$$

The ideal gas enthalpy is then obtained by integrating c_p in Equation (E1.6), i.e.

$$H_i^* = \int_0^T c_{pi}\, dT' \tag{E1.9}$$

Note that Equation (E1.6) is of the form

$$c_{pi}^* = \sum_{j=0}^{2} c_{ij} T^j \tag{E1.10}$$

and therefore H_i^* in Equation (E1.10) is of the form

$$H_i^* = \sum_{j=1}^{3} h_{ij} T^j \tag{E1.11}$$

Equation (E1.11) is similar to Equation (E1.4) with $a_{0i} = a_{4i} = a_{5i}$.

Heat Loss Calculation

The heat loss to the overburden and underburden is calculated using the method of Vinsome and Westerveld (1980). They assumed a temperature profile in the overburden and underburden of the form:

$$T(t,z) = (\theta - \theta^0 + b_1 z + b_2 z^2) \exp(- z / d) + \theta^0) \quad \text{(E1.12)}$$

where

$T(t,z$	=	over/underburden temperature at time t at a distance z from the reservoir boundary
b_1, b_2	=	time-dependent parameters
d	=	thermal diffusion length
θ	=	temperature in the boundary grid block
θ^0	=	initial temperature in boundary grid block

The diffusion length is taken as:

$$d = \frac{\sqrt{\eta t}}{2} \quad \text{(E1.13)}$$

where η is the thermal diffusivity

$$\eta = \frac{\kappa_R}{c_R \tilde{\rho}_R} \quad \text{(E1.14)}$$

and where

c_R	=	rock heat capacity
$\tilde{\rho}_R$	=	mass density of rock
κ_R	=	rock thermal conductivity

Vinsome and Westerveld (1980) derived the following expression for b_1 and b_2 and the heat loss rate:

$$b_1^{n+1} = \frac{\dfrac{\eta \Delta t \left(\theta^{n+1} - \theta^0\right)}{d^{n+1}} + \xi^n - \dfrac{\left(d^{n+1}\right)^3 \left(\theta^{n+1} - \theta^n\right)}{\eta \Delta t}}{3\left(d^{n+1}\right)^2 + \eta \Delta t} \quad \text{(E1.15)}$$

$$b_2^{n+1} = \frac{2 b_1^{n+1} \left(d^{n+1}\right) - \left(\theta^{n+1} - \theta^0\right) + \dfrac{\left(d^{n+1}\right)^2 \left(\theta^{n+1} - \theta^n\right)}{\eta \Delta t}}{2\left(d^{n+1}\right)^2} \quad \text{(E1.16)}$$

$$\xi^n = \left[\left(\theta - \theta^0\right) d + b_1 d^2 + 2 b_2 d^3\right]^n \quad \text{(E1.17)}$$

$$Q_{loss} = \kappa_R A \left[\frac{\left(\theta^{n+1} - \theta^0\right)}{d^{n+1}} - b_1^{n+1} \right] \tag{E1.18}$$

where A is the cross-sectional area for heat loss to the overburden/underburden.

For further information, please refer to Kenyon and Behie (1987), Peaceman (1983), Peaceman (1978), Peng and Robinson (1976), and Soave (1972).